HW 9

(Q-1) For a patch $x(r,s) = (r, s, f(r,s))$, prove the area is given by
$$\iint \sqrt{1 + f_r^2 + f_s^2}\, dr ds.$$
(Q-2) Using (Q-1), calculate the area of the hemisphere $x(r,s) = (r, s, \sqrt{1 - r^2 - s^2})$.
(Q-3) Prove that the u^i curves are lines of curvature if and only if $g_{12} \equiv L_{12} \equiv 0$.
(Q-4) Prove that the circles of latitude and the meridians of a surface of revolution are lines of curvature.
(Q-5) Suppose $g_{12} \equiv 0$. Prove
$$K = -\frac{1}{2\sqrt{g}}\Big(\frac{\partial}{\partial u^2}\big(\frac{\partial g_{11}}{\partial u^2}/\sqrt{g}\big) + \frac{\partial}{\partial u^1}\big(\frac{\partial g_{22}}{\partial u^1}/\sqrt{g}\big)\Big).$$
(Q-6) Suppose $g_{12} \equiv 0, g_{11} \equiv 1$. Prove
$$K\sqrt{g_{22}} + \frac{\partial^2 \sqrt{g_{22}}}{(\partial u^1)^2} = 0.$$
(Q-7) Determine which points on the patch $x(u,v) = (u, v, u^3 + v^3)$ have $K > 0$ and which have $K < 0$?
(Q-8) Prove $n_1 \times n_2 = K\sqrt{g}n$.
(Q-9) What are the principal curvatures of a surface of revolution?
(Q-10) Let $\gamma(s)$ be a unit speed curve on a surface M. Its geodesic torsion is $\tau_g = \langle S, dn/ds \rangle$, where $S = n \times T$. Prove $\tau_g \equiv 0$ if and only if γ is a line of curvature.