

HW 8

(Q-1) Show that for $M = S^2$, the Weingarten map $L = \pm \text{Id}$ by computing L_{ik} in a coordinate patch and raising an index.

(Q-2) Show that for $M = S^1 \times (0, 1)$, L can be represented by the matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

(Q-3) Find L for the torus from (HW-4, Q-1).

(Q-4) Prove that $II(X, Y) = \langle L(X), Y \rangle = \langle X, L(Y) \rangle$ for all $X, Y \in T_p M$.

(Q-5) Find the matrix (L_j^i) for a surface of revolution.

(Q-6) Let $f: V \rightarrow U$ be a change of coordinates. How are the \bar{L}_β^α related to the L_j^i ?

(Q-7) Let $\gamma(s) = x(\gamma^1(s), \gamma^2(s))$ be a unit speed curve on a surface. Note (T, S, n) gives a positive orthonormal frame, with $n(s) = n(\gamma^1(s), \gamma^2(s))$ viewed as a function of s . Prove the following analogues of the Frenet-Serret equations:

$$T' = II(T, T)n + \kappa_g S$$

$$S' = -\kappa_g T + II(T, S)n$$

$$n' = -II(T, T)T - II(T, S)S.$$

(Q-8) Find the Gaussian and mean curvatures of the plane, sphere, and the torus.

(Q-9) Prove $H^2 \geq K$. When does equality hold?

(Q-10) Let $X, Y \in T_p M$ be orthonormal vectors. Prove

$$H = \frac{1}{2}(II(X, X) + II(Y, Y)).$$