

HW 4

(Q-1) Define

$$x(u^1, u^2) = ((2 + \cos u^1) \cos u^2, (2 + \cos u^1) \sin u^2, \sin u^1)$$

with $-\pi < u^1, u^2 < \pi$. Prove x is a simple surface. Compute x_1, x_2, n as functions of u^1, u^2 .

(Q-2) Let $\alpha(t) = (r(t), z(t))$ be a regular injective curve with $r(t) > 0$ for all t , and consider

$$x(t, \theta) = (r(t) \cos \theta, r(t) \sin \theta, z(t)),$$

with $-\pi < \theta < \pi$. Prove it is a simple surface. Compute x_1, x_2, n .

(Q-3) The surface from (Q-2) is called a surface of revolution, and the t -curves are called meridians, and the θ -curves are called circles of latitudes, and the z -axis the axis of revolution. Prove the surface from (Q-1) is a surface of revolution, and compute its meridians and latitudes.

(Q-4) Let $U = \{(u^1, u^2) \mid 0 < u^1, 0 < u^2 < 2\pi\}$ and

$$x(u^1, u^2) = (u^1 \cos u^2, u^1 \sin u^2, u^1 + u^2).$$

Prove x is a simple surface.

(Q-5) Let $\theta(u^1)$ be a C^k function. Prove

$$x(u^1, u^2) = (u^2 \cos \theta(u^1), u^2 \sin \theta(u^1), u^1)$$

is a simple surface.

(Q-6) Let $x(u^1, u^2) = (u^1 + u^2, u^1 - u^2, u^1 u^2)$. Show x is a simple surface. Find the normal and the equation of the tangent plane at $u^1 = 1, u^2 = 2$.

(Q-7) Let

$$x(\theta, v) = (\cos \theta, \sin \theta, 0) + v(\sin \frac{\theta}{2} \cos \theta, \sin \frac{\theta}{2} \sin \theta, \cos \frac{\theta}{2})$$

with $\theta \in (-\pi, \pi), v \in (-\frac{1}{2}, \frac{1}{2})$. Compute $n(\theta, 0)$ and show that

$$\lim_{\theta \rightarrow -\pi} n(\theta, 0) = - \lim_{\theta \rightarrow \pi} n(\theta, 0)$$

while

$$\lim_{\theta \rightarrow -\pi} x(\theta, 0) = \lim_{\theta \rightarrow \pi} x(\theta, 0).$$

Draw the surface.

(Q-8) Let $S^2 = \{(u, v, w) \in \mathbb{R}^3 \mid u^2 + v^2 + w^2 = 1\}$ and $\mathbb{R}^2 = \{(u, v, w) \in \mathbb{R}^3 \mid w = 0\}$. The line joining $(u, v, 0)$ and $(0, 0, 1)$ intersects S^2 is a point other than $(0, 0, 1)$; call this point $x(u, v)$ and prove $x: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a simple surface. (The inverse mapping is called the stereographic projection.)

(Q-9) Let

$$x(\theta, \phi) = (\cos \phi \cos \theta, 2 \cos \phi \sin \theta, 3 \sin \phi), \quad \theta \in (-\pi, \pi), \phi \in (-\pi/2, \pi/2).$$

Show x is a simple surface. What is it?

(Q-10) Let

$$x(u, v) = (\sqrt{1 - u^2} \cos v, \sqrt{1 - u^2} \sin v, u), \quad u \in (-1, 1), v \in (-\pi, \pi).$$

Show x is a simple surface. What is it?