

HW 3

- (Q-1) Compute the Frenet-Serret apparatus for the non-unit speed curve $\beta(t) = (t, t^2, t^3)$.
- (Q-2) Find the curvature and torsion for the non-unit speed curve $(e^t \cos t, e^t \sin t, e^t)$.
- (Q-3) Compute $\int_C (xy + 1)dx + (x^2/y)dy$ along each of the following curves:
- C is given by $\alpha(t) = (t, t)$, $0 \leq t \leq 1$.
 - C is given by $\beta(t) = (t^2, t^3)$, $0 \leq t \leq 1$.
- (Q-4) Compute $\int_C (xdy - ydx)/(x^2 + y^2)$ where C is the straight line segment:
- from $(0, 1)$ to $(1, 0)$;
 - from $(1, 0)$ to $(1, 1)$;
 - from $(1, 1)$ to $(0, 1)$;
- (Q-5) Verify Green's theorem for $\int_C (xdy - ydx)/(x^2 + y^2)$ where C is the triangle with vertices $(0, 1)$, $(1, 0)$, $(1, 1)$.
- (Q-6) For a unit speed plane curve α , prove that $\kappa(s) = |k_P(s)|$, and if $\kappa(s) \neq 0$, then $n(s) = \pm N(s)$, and $n'(s) = -k_P(s)T(s)$, where k_P is the planar curvature.
- (Q-7) If $\beta(t)$ is a closed curve with period a , and $\alpha(s)$ is its arc-length reparametrization, then prove that $\alpha(s)$ is closed with period $L = \int_0^a |d\beta/dt|dt$.
- (Q-8) If α is a unit speed closed plane curve with period L , prove that there is a well-defined function $\theta: [0, L] \rightarrow \mathbb{R}$, such that $T(s) = (\cos \theta(s), \sin \theta(s))$.
- (Q-9) If α is a simple closed unit-speed plane curve, prove that tangent direction $T: [0, L] \rightarrow S^1$ is an onto map.
- (Q-10) Give an example of a non-simple closed unit-speed plane curve where the tangent direction $T: [0, L] \rightarrow S^1$ is not an onto map.