1. **Likelihood ratios**. Suppose we want to test whether a given coin is fair or unfair. That is, given a sample $X_1, \ldots, X_n$, each Bernoulli distributed with probability p of success, we want to test the null hypothesis $p = 1/2$ against the alternate hypothesis $p = q$, for some $q \neq 1/2$.

   (a) Argue that $Y = \sum X_I$ is a sufficient statistic for this model, and that Y is binomially distributed. So it's enough to look at Y.

   (b) Find the likelihood ratio function.

   (c) Show that the most powerful critical region is of the form $Y \geq C$ for q greater than $1/2$, and $Y \leq C$ for q less than $1/2$, where C is a constant potentially depending on α, n, q. [Hint - it may be helpful to write the likelihood function as $A^x B^n$ for some A, B.]

   (d) Actually, C does not depend on q if we restrict ourself to the case where q if greater than $1/2$. Why is this? This shows that, in that case, this estimator is actually uniformly most powerful