1. **Bayesian inference**. Suppose X_i are a sample of size n from $\mathrm{Unif}[0, \Theta]$, where our prior distribution for Θ is exponential with parameter λ. Show that the posterior distribution of Θ is

$$\max x_i + \mathrm{Exp}(\lambda).$$

2. **Rao-Blackwell**. Last week we briefly covered this example - let's do it fully. Suppose X_i are a sample of size n from $\mathrm{Unif}[0, \theta]$. Then we can let $T = X_{(n)} = \max X_i$. Define Y to be $2\bar{X}$. (Here T happens to be the MLE, and Y the method-of-moments estimator).

   - Show that T is sufficient.
   - Find $\mathbb{E}[Y|T]$. (Note: the conditional distribution of X_i on T is not uniform.
   - Compare $\mathrm{Var}\, Y$ and $\mathrm{Var}(\mathbb{E}[Y|T])$. Note: we know that T is $\mathrm{Beta}(n, 1)$-distributed with variance

$$\frac{n^2}{(n+1)^2(n+2)}.$$