1. Introduction: The limit and how it can fail to exist.
2. Example (to do at the board): Investigate the continuity and the existence of limits for the function

$$f(x) = \frac{2x - 6}{x^2 - 4x + 3}$$

3. Example (for you to work on): Consider the function

$$f(x) = \frac{|x|}{x}. \quad (x \neq 0)$$

   (a) Draw a graph of this function. (Hint: consider the cases $x > 0$ and $x < 0$ separately).

   (b) Where is this function continuous?

   Note that $f(0)$ is not defined.

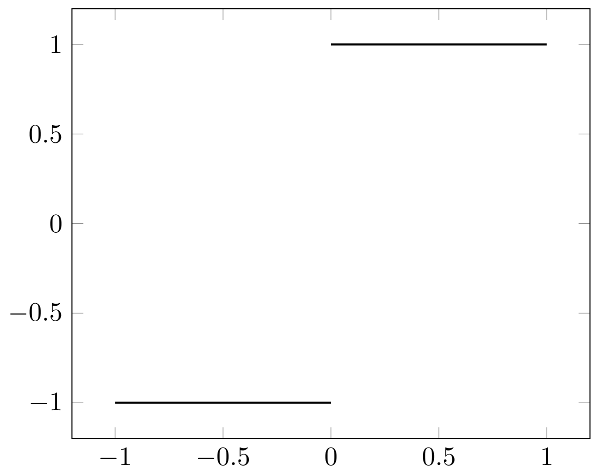
   (c) Can we define $f(0)$ in some way to make the function continuous at zero? Find such a number or explain why it cannot be done.

   (d) Can we define $f(0)$ in some way to make the function left continuous at zero? Find such a number or explain why it cannot be done.

   (e) Can we define $f(0)$ in some way to make the function right continuous at zero? Find such a number or explain why it cannot be done.

Solutions for (3)

When $x > 0$, $|x| = x$, so we get $f(x) = x/x = 1$. On the other hand, if x is negative, then $|x| = -x$, so $f(x) = -x/x = -1$. So here's a graph of the function in question:



(Note that the function is not defined at zero).

So f is continuous everywhere except at zero (since for any other point, f is locally equal to either the constant function -1 or 1, which is continuous). It can't be continuous at zero since it's not even defined there.

If we want to make it continuous, we have to look at the left and right limits of f:

$$\lim_{x\to 0^-} f(x) = -1$$
$$\lim_{x\to 0^+} f(x) = 1$$

Since these are different, we can't make the function continuous; but we could make it left continuous by setting $f(0) = -1$ and right continuous by setting $f(0) = 1$.