True or False? (Note: for all of these, we can assume that we are in a finite-dimensional vector space over the reals).

1. If a set of two vectors $\{v_1, v_2\}$ is linearly dependent, then v_2 must be a multiple of v_1.
2. A linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^6$ cannot be onto.
3. If n-by-n matrices A and B are similar, and A is invertible, then B is invertible.
4. Any ten vectors in $\mathbb{R}^4$ must be a spanning set.
5. If matrices A and B commute, then they must be square matrices.
6. Matrices A and B commute if and only if $A = B^{-1}$.
7. Any subspace of $\mathbb{R}^n$ has a basis.
8. It is possible for a subspace of $\mathbb{R}^n$ to have exactly two bases.
9. It is possible for a subspace of $\mathbb{R}^n$ to have exactly two orthonormal bases.
10. There exist a, b, and c to make the following set of vectors a basis for $\mathbb{R}^3$:

$$\left\{ \begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ b \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ c \\ 1 \end{bmatrix} \right\}$$

11. If T and S are both one-to-one, then so is TS. (You can assume TS is well-defined).
12. If T and S are both onto, then so is TS. (You can assume TS is well-defined).
13. For any invertible matrices A and B, we have $(AB)^{-1} = A^{-1}B^{-1}$.
14. The set of real-coefficient polynomials of degree 7 is a vector space.
15. Any linear transformation from $\mathbb{R}^6$ to $\mathbb{R}^{10}$ is one-to-one.
16. Any set containing the zero vector is linearly dependent.
17. Given k vectors in $\mathbb{R}^n$, their span is a subspace of dimension k.
18. For a matrix A, the solutions of the equation $A(x) = b$ constitute a subspace.

    (*Note: If you would like to check your work, the sum of the True statements' numbers is 65.*)

1. False: try letting v_1 be the zero vector.
2. True: its rank is at most 4, so the output can't be $\mathbb{R}^6$.
3. True: if $A = SBS^{-1}$, then $ASB^{-1}S^{-1} = I$, so A is invertible.
4. False: consider 10 copies of the zero vector.
5. True: suppose A is nxm. Then in order for B to multiply both ways, B is mxn. So AB is nxn and BA is mxm. For them to be the same, $m = n$. So A is square.
6. False: the zero matrix commutes with itself.
7. True: just keep adding independent vectors until you get a basis.
8. False: if v_1 is in the basis, can replace it with $2v_1$, $3v_1$, and so on.
9. True: pick your favorite 1D subspace.
10. False: that matrix isn't invertible since the first and last rows are the same.
11. True: For any $x \neq y$, $Sx \neq Sy$, so $TSx \neq TSy$.
12. True: For any b there exists some x such that $Sx = b$. Then there's some y such that $Ty = b$. So $STy = Sx = b$.
13. False: it's $B^{-1}A^{-1}$.
14. False: x^7 and $-x^7$ are in the space, but their sum is not.
15. False: consider the zero transformation.
16. True: the zero vector is dependent (see HW) and so any set containing it is.
17. False: consider k copies of the zero vector.
18. False: 0 isn't in it (unless $b = 0$).