MATH 54 SUMMER 2017, QUIZ 9

Mark each of the following true or false and give a short explanation.

(a) The set of nonnegative real numbers (i.e. $[0, \infty)$) is a subspace of $\mathbb{R}$.

False. It is not closed under scalar multiplication. For instance, $5 \in [0, \infty)$ but $-1 \cdot 5 = -5 \notin [0, \infty)$.

(b) $\{\mathbf{0}\}$ is a subspace of $\mathbb{R}^n$.

True.

① $\vec{0} \in \{\vec{0}\}$

② If $\vec{u}, \vec{v} \in \{\vec{0}\}$ then $\vec{u} = \vec{0}$ and $\vec{v} = \vec{0}$ so $\vec{u} + \vec{v} = \vec{0} + \vec{0} = \vec{0} \in \{\vec{0}\}$.

③ If $\vec{u} \in \{\vec{0}\}$ and $c \in \mathbb{R}$ then $\vec{u} = \vec{0}$ so $c\vec{u} = c\vec{0} = \vec{0} \in \{\vec{0}\}$.

(c) There are vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ in $\mathbb{R}^3$ such that ~~span~~$\{\mathbf{v}_1, \mathbf{v}_2, 3\mathbf{v}_1 + \mathbf{v}_2\}$ is a basis for $\mathbb{R}^3$.

False. The vectors $\vec{v}_1, \vec{v}_2, 3\vec{v}_1 + \vec{v}_2$ can never be linearly independent because

$$3 \cdot \vec{v}_1 + \vec{v}_2 - 1 \cdot (3\vec{v}_1 + \vec{v}) = \vec{0}.$$

(d) The following vectors are a basis for $\mathbb{R}^3$:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

False. Two vectors cannot span all of $\mathbb{R}^3$ because the corresponding matrix cannot have a pivot in every row.

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