

MATH 54 SUMMER 2017, QUIZ 6

Suppose A is a 5×5 matrix such that the solution set of $Ax = 0$ is $\text{span}\{a, b, c\}$ and $Ad = f$. Find all solutions to $Ax = f$.

$$a = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \quad f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Since \vec{d} is a solution to $A\vec{x} = \vec{f}$, the set of all solutions to $A\vec{x} = \vec{f}$ is the set of vectors of the form $\vec{d} + \vec{v}$ where \vec{v} is any solution to $A\vec{x} = \vec{0}$. In other words, the set of solutions to $A\vec{x} = \vec{f}$ is

$$\left\{ \vec{d} + s\vec{a} + t\vec{b} + r\vec{c} \mid s, t, r \in \mathbb{R} \right\}$$

$$= \left\{ \begin{bmatrix} 1+s+t+r \\ 2+t+r \\ 3+r \\ 4 \\ 5 \end{bmatrix} \mid s, t, r \in \mathbb{R} \right\}$$

either form is acceptable