Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 5x_2 - 2x_3 \\ 5x_2 + x_3 \end{bmatrix}.$$

$T$ is a linear transformation (you do not have to check this). Find the standard matrix of $T$ and find a vector $\mathbf{v} \in \mathbb{R}^3$ such that

$$T(\mathbf{v}) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

so the standard matrix of T is

$$\begin{bmatrix} 1 & 5 & -2 \\ 0 & 5 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 5 & -2 & 5 \\ 0 & 5 & 1 & 4 \end{array}\right] \longrightarrow$$

↳ already in REF

$x_3$ is free

$$5x_2 + x_3 = 4$$
$$x_1 + 5x_2 - 2x_3 = 5$$

$\Rightarrow$

$x_3$ is free

$$x_2 = \frac{4}{5} - \frac{x_3}{5}$$
$$x_1 = 5 + 2x_3 - 5x_2$$
$$= 1 + 3x_3$$

Setting $x_3$ to $0$ gives: $x_1 = 1, \ x_2 = \frac{4}{5}, \ x_3 = 0$

So $\boxed{T\left(\begin{bmatrix} 1 \\ 4/5 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 4 \end{bmatrix}}$

<u>Check:</u> $T\left(\begin{bmatrix} 1 \\ 4/5 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 + 5(4/5) - 2(0) \\ 5(4/5) + 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

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