

MATH 54 SUMMER 2017, QUIZ 4

Are the following vectors linearly dependent? If not, explain why not. If so, find a nontrivial linear combination of them that is equal to $\mathbf{0}$ (i.e. find real numbers a , b and c , not all zero, such that $a\mathbf{u} + b\mathbf{v} + c\mathbf{w} = \mathbf{0}$).

$$\mathbf{u} = \begin{bmatrix} 4 \\ 5 \\ -2 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} -2 \\ 6 \\ 3 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 3 \\ 5 & 6 & 1 \\ -2 & 3 & -2 \end{bmatrix} \xrightarrow{R_2 = R_2 - \frac{5}{4}R_1} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 17/2 & -11/4 \\ -2 & 3 & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 + \frac{1}{2}R_1} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 17/2 & -11/4 \\ 0 & 2 & -1/2 \end{bmatrix}$$

$$\xrightarrow{R_3 = R_3 - \frac{4}{17}R_2} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 17/2 & -11/4 \\ 0 & 0 & -1/2 + \frac{11}{17} \end{bmatrix}$$

The vectors are not linearly dependent because the above matrix has a pivot in every column.