

MATH 54 SUMMER 2017, QUIZ 3

Express  $w$  as a linear combination of  $u$  and  $v$  (i.e. find real numbers  $a$  and  $b$  such that  $au + bv = w$ ).

$$u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad v = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad w = \begin{bmatrix} 7 \\ 10 \\ 11 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 1 & -1 & 7 \\ 2 & 0 & 10 \\ 3 & 2 & 11 \end{array} \right] \xrightarrow{R_2 = R_2 - 2R_1} \left[ \begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 2 & -4 \\ 3 & 2 & 11 \end{array} \right]$$

$$\xrightarrow{R_3 = R_3 - 3R_1} \left[ \begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 2 & -4 \\ 0 & 5 & -10 \end{array} \right]$$

$$\xrightarrow{R_2 = \frac{1}{2}R_2} \left[ \begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 1 & -2 \\ 0 & 5 & -10 \end{array} \right]$$

$$\xrightarrow{R_3 = R_3 - 5R_2} \left[ \begin{array}{cc|c} 1 & -1 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow \begin{array}{l} x_2 = -2 \\ x_1 - x_2 = 7 \end{array} \Rightarrow \begin{array}{l} x_2 = -2 \\ x_1 = 7 + x_2 = 5 \end{array}$$

$$\boxed{\vec{w} = 5\vec{u} - 2\vec{v}}$$

Check:

$$5 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5+2 \\ 10-0 \\ 15-4 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ 11 \end{bmatrix}$$