MATH 54 SUMMER 2017, QUIZ 29

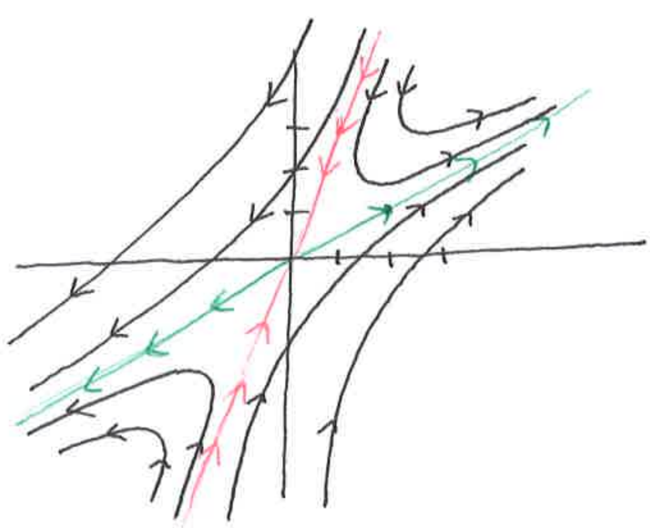
Suppose A is a 2×2 matrix and that $\mathbf{v}_1$ is an eigenvector of A with eigenvalue 2 and $\mathbf{v}_2$ is an eigenvector of A with eigenvalue -2.

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

(a) Find the general solution to $\mathbf{y}' = A\mathbf{y}$

$$\boxed{c_1 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}}$$

(b) Draw a picture of all solutions to $\mathbf{y}' = A\mathbf{y}$. Make sure to include the solutions that always stay in some eigenspace of A.



The solutions in green are solutions of the form $c e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. The always stay in the eigenspace E_2 of A and as $t \to \infty$ they rush away from the origin.

The solutions in red are solutions of the form $c e^{-2t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. They always stay in the eigenspace E_{-2} of A and as $t \to \infty$ they get closer to the origin.

The solutions in black are linear combinations of the green and red solutions. As $t \to \infty$ the green component gets larger and the red component gets smaller until the green component dominates.