MATH 54 SUMMER 2017, QUIZ 28

Solve the following initial value problem.

$$\mathbf{y}'(t) = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \mathbf{y}(t); \quad \mathbf{y}(0) = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

<u>Eigenvalues:</u>

$$\det \begin{bmatrix} 1-\lambda & 2 \\ 1 & 2-\lambda \end{bmatrix} = (1-\lambda)(2-\lambda) - 2 = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

Eigenvalues: $0, 3$

<u>Eigenvector for 0:</u>

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \xrightarrow{R2 = R2 - R1} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$x_1 + 2x_2 = 0$
x_2 free $\Rightarrow$ $x_1 = -2x_2$

$$\boxed{\begin{bmatrix} -2 \\ 1 \end{bmatrix}}$$

<u>Eigenvector for 3:</u>

$$\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \xrightarrow{R1 = \frac{1}{2}R1} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \xrightarrow{R2 = R2 + R1} \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$-x_1 + x_2 = 0$
x_2 free $\Rightarrow$ $x_1 = +x_2$

$$\boxed{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

<u>General Solution:</u>

$$y(t) = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

<u>IVP:</u>

$$\begin{bmatrix} 11 \\ 2 \end{bmatrix} = y(0) = c_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -2 & 1 & 11 \\ 1 & 1 & 2 \end{array}\right] \xrightarrow{\text{swap } R_1, R_2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ -2 & 1 & 11 \end{array}\right] \xrightarrow{R2 = R2 + 2R1} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 3 & 15 \end{array}\right] \xrightarrow{R2 = \frac{1}{3}R2} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R1 = R1 - R2} \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 5 \end{array}\right] \Rightarrow \begin{array}{l} c_1 = -3 \\ c_2 = 5 \end{array}$$

$$\boxed{y(t) = -3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + 5e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

<u>Check:</u>

$$y(0) = \begin{bmatrix} 6 \\ -3 \end{bmatrix} + \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix}$$

$$y'(t) = 15e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Ay(t) = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -3 \end{bmatrix} + 5e^{3t} \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 5e^{3t} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 15e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$