MATH 54 SUMMER 2017, QUIZ 26

Find the general solution of the following ODE.

$$y'' + 4y' + 4y = 2t^2$$

<u>Homogeneous Equation:</u> $y'' + 4y' + 4y = 0$

Auxiliary Equation: $r^2 + 4r + 4 = 0$

$$(r+2)^2 = 0$$

roots: -2 with multiplicity 2

General sol'n to homogeneous equation: $c_1 e^{-2t} + c_2 t e^{-2t}$

<u>Nonhomogeneous</u>: Guess $y(t) = At^2 + Bt + C$

$$y'(t) = 2At + B$$

$$y''(t) = 2A$$

$$2A + 4(2At + B) + 4(At^2 + Bt + C) = 2t^2$$

$$\Rightarrow \begin{array}{l} 4A = 2 \\ 8A + 4B = 0 \\ 2A + 4B + 4C = 0 \end{array} \Longrightarrow \begin{array}{l} A = \frac{1}{2} \\ B = -\frac{8}{4}A = -1 \\ C = -\frac{2}{4}A - \frac{4}{4}B = -\frac{1}{4} + 1 = \frac{3}{4} \end{array}$$

$$y(t) = \frac{1}{2}t^2 - t + \frac{3}{4}$$

General Sol'n to Nonhomogeneous equation: $\boxed{\frac{1}{2}t^2 - t + \frac{3}{4} + c_1 e^{-2t} + c_2 t e^{-2t}}$

<u>Check:</u> If $y(t) = \frac{1}{2}t^2 - t + \frac{3}{4}$ then $y'' + 4y' + 4y = 1 + 4(t-1) + 4(\frac{1}{2}t^2 - t + \frac{3}{4})$

$$= 1 + 4t - 4 + 2t^2 - 4t + 3$$

$$= 2t^2$$

If $y(t) = e^{-2t}$ then $y'' + 4y' + 4y = 4e^{-2t} + 4(-2e^{-2t}) + 4e^{-2t} = 0$

If $y(t) = te^{-2t}$ then $y'' + 4y' + 4y = 4te^{-2t} - 2e^{-2t} - 2e^{-2t} + 4(-2te^{-2t} + e^{-2t}$

$+ 4te^{-2t} = 0$