MATH 54 SUMMER 2017, QUIZ 24

Find a solution to the following initial value problem.

$$y'' + 6y' + 8y = 0; \quad y(0) = 1; \quad y'(0) = -8$$

Auxiliary equation: $r^2 + 6r + 8 = 0$

$(r+4)(r+2) = 0$

Roots: $-4, -2$

General Solution: $y(t) = c_1 e^{-4t} + c_2 e^{-2t}$

Initial Values:

$$1 = y(0) = c_1 e^{-4\cdot 0} + c_2 e^{-2\cdot 0} = c_1 + c_2$$

$$-8 = y'(0) = -4c_1 e^{-4\cdot 0} + (-2c_2 e^{-2\cdot 0}) = -4c_1 - 2c_2$$

Solve for c_1, c_2:

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -4 & -2 & -8 \end{array}\right] \xrightarrow{R2 = R2 + 4R1} \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 2 & -4 \end{array}\right] \xrightarrow{R1 = R1 - \frac{1}{2}R2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 2 & -4 \end{array}\right]$$

So $c_1 = 3$, $2c_2 = -4 \Rightarrow$ $c_1 = 3$, $c_2 = -2$

So a solution is:

$$\boxed{y(t) = 3e^{-4t} - 2e^{-2t}}$$

Check:

$y'(t) = -12e^{-4t} + 4e^{-2t}$ $\qquad$ $y(0) = 3e^{-4\cdot 0} - 2e^{-2\cdot 0} = 3 - 2 = 1$

$y''(t) = 48e^{-4t} - 8e^{-2t}$ $\qquad$ $y'(0) = -12e^{-4\cdot 0} + 4e^{-2\cdot 0} = -12 + 4 = -8$

$$y''(t) + 6y'(t) + 8y(t) = 48e^{-4t} - 8e^{-2t} + 6(-12e^{-4t} + 4e^{-2t}) + 8(3e^{-4t} - 2e^{-2t})$$

$$= 0$$

Date: July 28, 2017.