MATH 54 SUMMER 2017, QUIZ 21

Find an orthgonal basis for span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

$$\mathbf{v}_1 = \begin{bmatrix}1\\2\\3\\1\end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix}3\\5\\5\\2\end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix}10\\4\\8\\3\end{bmatrix}$$

<u>Use Gram-Schmidt:</u>

$$u_1 = v_1$$

$$u_2 = v_2 - \text{proj}_{\text{span}\{u_1\}}(v_2) = v_2 - \left(\frac{v_2 \cdot u_1}{u_1 \cdot u_1} u_1\right)$$

$$= v_2 - \left(\frac{30}{15} u_1\right) = \begin{bmatrix}3\\5\\5\\2\end{bmatrix} - 2\begin{bmatrix}1\\2\\3\\1\end{bmatrix} = \begin{bmatrix}1\\1\\-1\\0\end{bmatrix}$$

$$u_3 = v_3 - \text{proj}_{\text{span}\{u_1, u_2\}}(v_3) = v_3 - \left(\frac{v_3 \cdot u_1}{u_1 \cdot u_1} u_1 + \frac{v_3 \cdot u_2}{u_2 \cdot u_2} u_2\right)$$

$$= \begin{bmatrix}10\\4\\8\\3\end{bmatrix} - \left(\frac{45}{15}\begin{bmatrix}1\\2\\3\\1\end{bmatrix} + \frac{6}{3}\begin{bmatrix}1\\1\\-1\\0\end{bmatrix}\right) = \begin{bmatrix}10-3-2\\4-6-2\\8-9+2\\3-3\end{bmatrix} = \begin{bmatrix}5\\-4\\1\\0\end{bmatrix}$$

$$v_2 \cdot u_1 = \begin{bmatrix}3\\5\\5\\2\end{bmatrix} \cdot \begin{bmatrix}1\\2\\3\\1\end{bmatrix} = 3+10+15+2 = 30$$

$$u_1 \cdot u_1 = \begin{bmatrix}1\\2\\3\\1\end{bmatrix} \cdot \begin{bmatrix}1\\2\\3\\1\end{bmatrix} = 1^2+2^2+3^2+1^2 = 1+4+9+1 = 15$$

$$v_3 \cdot u_1 = \begin{bmatrix}10\\4\\8\\3\end{bmatrix} \cdot \begin{bmatrix}1\\2\\3\\1\end{bmatrix} = 10+8+24+3 = 45$$

$$v_3 \cdot u_2 = \begin{bmatrix}10\\4\\8\\3\end{bmatrix} \cdot \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} = 10+4-8 = 6$$

$$u_2 \cdot u_2 = \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} \cdot \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} = 1+1+(-1)(-1) = 3$$

So an orthogonal basis is $\boxed{\left\{\begin{bmatrix}1\\2\\3\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\\0\end{bmatrix}, \begin{bmatrix}5\\-4\\1\\0\end{bmatrix}\right\}}$

Date: July 23, 2017.

<u>Check:</u>

$$u_1 \cdot u_2 = \begin{bmatrix}1\\2\\3\\1\end{bmatrix} \cdot \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} = 1+2-3 = 0 \qquad u_2 \cdot u_3 = \begin{bmatrix}1\\1\\-1\\0\end{bmatrix} \cdot \begin{bmatrix}5\\-4\\1\\0\end{bmatrix}$$

$$u_1 \cdot u_3 = \begin{bmatrix}1\\2\\3\\1\end{bmatrix} \cdot \begin{bmatrix}5\\-4\\1\\0\end{bmatrix} = 5-8+3 = 0 \qquad = 5-4-1 = 0$$