

MATH 54 SUMMER 2017, QUIZ 20

Express y as a linear combination of u , v , and w . Do not use row reduction. You may assume without checking that $\{u, v, w\}$ is an orthogonal set. Make sure to show your work.

$$u = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 5 \\ 2 \\ -3 \\ -8 \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ -2 \\ 7 \\ 9 \end{bmatrix}$$

$$\vec{u} \cdot \vec{y} = 3(-1) + 2(-2) + 1 \cdot 7 + 2 \cdot 9 = 18 \quad \vec{u} \cdot \vec{u} = 3^2 + 2^2 + 1^2 + 2^2 = 18$$

$$\vec{v} \cdot \vec{y} = 5(-1) + 2(-2) + (-3) \cdot 7 + (-8)(9) = -102 \quad \vec{v} \cdot \vec{v} = 5^2 + 2^2 + (-3)^2 + (-8)^2 = 102$$

$$\vec{w} \cdot \vec{y} = 1 \cdot (-1) + (-2)(-2) + 3(7) + (-1) \cdot 9 = 15 \quad \vec{w} \cdot \vec{w} = 1^2 + (-2)^2 + 3^2 + (-1)^2 = 15$$

$$\text{So } \vec{y} = \frac{u \cdot y}{u \cdot u} \vec{u} + \frac{v \cdot y}{v \cdot v} \vec{v} + \frac{w \cdot y}{w \cdot w} \vec{w}$$

$$= \boxed{\vec{u} - \vec{v} + \vec{w}}$$

Check:

$$\begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \\ -3 \\ -8 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 3-5+1 \\ 2-2-2 \\ 1+3+3 \\ 2+8-1 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 7 \\ 9 \end{bmatrix}$$