MATH 54 SUMMER 2017, QUIZ 19

$$\mathbf{u} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 2 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 5 \\ 1 \\ -1 \\ -8 \end{bmatrix}$$

(a) Find the length of $\mathbf{u}$.

$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{3^2+2^2+1^2+2^2} = \sqrt{9+4+1+4} = \boxed{\sqrt{18}}$

(b) Find the distance between $\mathbf{u}$ and $\mathbf{v}$—i.e. find $\text{dist}(\mathbf{u}, \mathbf{v})$.

$\text{dist}(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\| = \left\| \begin{bmatrix} 2 \\ 2 \\ 2 \\ 0 \end{bmatrix} \right\| = \sqrt{2^2+2^2+2^2+0^2} = \boxed{\sqrt{12}}$

(c) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos\theta$

where θ is the angle between $\vec{u}$ & $\vec{v}$

$\|\vec{u}\| = \sqrt{18}$

$\|\vec{v}\| = \sqrt{1^2+0^2+(-1)^2+2^2} = \sqrt{6}$

$\vec{u} \cdot \vec{v} = 3 \cdot 1 + 2 \cdot 0 + 1 \cdot (-1) + 2 \cdot 2 = 6$

So $\boxed{\cos\theta = \frac{6}{\sqrt{18}\sqrt{6}}}$

(d) Find a unit vector in the same direction as $\mathbf{u}$.

$\frac{\vec{u}}{\|\vec{u}\|} = \boxed{\begin{bmatrix} 3/\sqrt{18} \\ 2/\sqrt{18} \\ 1/\sqrt{18} \\ 2/\sqrt{18} \end{bmatrix}}$

(e) Is any pair of the three vectors above orthogonal to each other?

$\vec{u} \cdot \vec{v} = 6$

$\vec{v} \cdot \vec{w} = 1 \cdot 5 + 0 \cdot 1 + (-1)(-1) + 2(-8) = -10$

$\vec{u} \cdot \vec{w} = 3 \cdot 5 + 2 \cdot 1 + 1 \cdot (-1) + 2 \cdot (-8) = 0$

Yes

$\vec{u}$ and $\vec{w}$ are orthogonal to each other

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