MATH 54 SUMMER 2017, QUIZ 18

Find matrices (possibly with complex entries) P and D such that D is diagonal, P is invertible and $A = PDP^{-1}$.

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

<u>Eigenvalues:</u>

$$\det[A - \lambda I] = \det\begin{bmatrix} 3-\lambda & -1 \\ 2 & 1-\lambda \end{bmatrix}$$

$$= (3-\lambda)(1-\lambda) + 2$$

$$= \lambda^2 - 4\lambda + 5$$

roots: $\dfrac{4 \pm \sqrt{16-20}}{2} = 2 \pm i$

<u>Eigenvectors:</u>

$2+i$: $A - (2+i)I = \begin{bmatrix} 1-i & -1 \\ 2 & -1-i \end{bmatrix} \xrightarrow{R2 = R2 - \frac{2}{1-i}R1} \begin{bmatrix} 1-i & -1 \\ 0 & 0 \end{bmatrix}$

$\dfrac{1}{1-i} = \dfrac{1+i}{1^2+1^2} = \dfrac{1+i}{2}$ so $-\dfrac{2}{1-i} = -(1+i)$

$(1-i)x_1 - x_2 = 0$
x_2 free
$\Rightarrow x_1 = \dfrac{1}{1-i}x_2 = \dfrac{1+i}{2}x_2$

So $\begin{bmatrix} (1+i)/2 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $2+i$

and $\begin{bmatrix} (1-i)/2 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $2-i$

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Therefore $A = PDP^{-1}$ where

$$\boxed{P = \begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2+i & 0 \\ 0 & 2-i \end{bmatrix}}$$