MATH 54 SUMMER 2017, QUIZ 16

Find the determinant of the following matrix:

$$\begin{bmatrix} 4 & 12 & 8 & 4 \\ 0 & 0 & 3 & 1 \\ 1 & 1 & -1 & 1 \\ 2 & 2 & 1 & 7 \end{bmatrix}$$

Let's solve this by row reduction (which is a bit easier than cofactor expansion).

$$\begin{bmatrix} 4 & 12 & 8 & 4 \\ 0 & 0 & 3 & 1 \\ 1 & 1 & -1 & 1 \\ 2 & 2 & 1 & 7 \end{bmatrix} \xrightarrow{R1=\frac{1}{4}R1} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 \\ 1 & 1 & -1 & 1 \\ 2 & 2 & 1 & 7 \end{bmatrix} \xrightarrow{R3=R3-R1} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & -2 & -3 & 0 \\ 2 & 2 & 1 & 7 \end{bmatrix}$$

$$\xrightarrow{R4=R4-2R1} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & -4 & -3 & 5 \end{bmatrix} \xrightarrow{R4=R4-2R3} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & 3 & 5 \end{bmatrix}$$

$$\xrightarrow{R4=R4-R2} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xrightarrow{\text{Swap } R2 \text{ and } R3} \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -2 & -3 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Let A refer to the original matrix and let R refer to the matrix in REF above. The only row operations above that changed the determinant were the first one, which multiplied the determinant by $1/4$ and the last, which multiplied the determinant by -1. All the other row operations were row replacements, which do not change the determinant. Therefore

$$\det(R) = -1/4 \det(A).$$

Since R is upper triangular, its determinant is the product of the entries on its diagonal:

$$\det(R) = 1 \cdot -2 \cdot 3 \cdot 4 = -24$$

Therefore

$$\det(A) = -4 \det(R) = -4(-24) = \boxed{96}.$$

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