MATH 54 SUMMER 2017, QUIZ 14

Suppose $\mathbf{v}$ is a vector in $\mathbb{R}^2$ such that $[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$, where $\mathcal{B}$ and $\mathcal{C}$ are the bases for $\mathbb{R}^2$ shown below (you do not need to check that they are bases for $\mathbb{R}^2$).

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\} \quad \mathcal{C} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

(a) What is $\mathbf{v}$?

Since

$$[\mathbf{v}]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

we know that

$$\mathbf{v} = -3 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} -6+6 \\ -12-2 \end{bmatrix} = \begin{bmatrix} 0 \\ -14 \end{bmatrix}$$

Another valid way to find the solution is to multiply $[\mathbf{v}]_{\mathcal{B}}$ by $\underset{\mathcal{E} \leftarrow \mathcal{B}}{P} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$.

(b) What is $[\mathbf{v}]_{\mathcal{C}}$?

We know $\mathbf{v}$ so to find $[\mathbf{v}]_{\mathcal{C}}$ we need to find real numbers a and b such that

$$\mathbf{v} = a \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

This amounts to solving the following system:

$$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 1 & 2 & -14 \end{array}\right] \xrightarrow{R1 = \frac{1}{2}R1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 2 & -14 \end{array}\right] \xrightarrow{R2 = R2 - R1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & -14 \end{array}\right]$$

$$\xrightarrow{R1 = R1 - R2} \left[\begin{array}{cc|c} 1 & 0 & 14 \\ 0 & 1 & -14 \end{array}\right]$$

Therefore $[\mathbf{v}]_{\mathcal{C}} = \begin{bmatrix} 14 \\ -14 \end{bmatrix}$.

Another perfectly acceptable way to find the solution was to compute $\underset{\mathcal{C} \leftarrow \mathcal{E}}{P}$ by inverting

$$\underset{\mathcal{E} \leftarrow \mathcal{C}}{P} = \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}$$

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Another perfectly acceptable way to find the solution was to compute $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ by row reducing

$$\left[\begin{array}{cc|cc} 2 & 2 & 2 & 3 \\ 1 & 2 & 4 & -1 \end{array}\right] \to \left[I_2 \mid \underset{\mathcal{C}\leftarrow\mathcal{B}}{P}\right]$$