MATH 54 SUMMER 2017, QUIZ 13

Let $T\colon \mathbb{P}_2 \to \mathbb{P}_1$ be the function defined by

$$T(p) = p(1)x + p(2).$$

(a) Find $T(1)$, $T(x)$, and $T(x^2)$.

$T(1) = x+1$

$T(x) = x+2$

$T(x^2) = x+4$

(b) Find the coordinate vectors relative to $\mathcal{C}$ of $T(1)$, $T(x)$, and $T(x^2)$.

$\mathcal{B} = \{1, x, x^2\}$

$\mathcal{C} = \{1, x\}$

$[T(1)]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$[T(x)]_{\mathcal{C}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$[T(x^2)]_{\mathcal{C}} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(c) T is a linear transformation (you do not have to check this). Find the matrix of T relative to the bases $\mathcal{B}$ and $\mathcal{C}$.

$${}_{\mathcal{C}}[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

(d) Is T one-to-one? Onto?

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R2 = R2 - R1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -3 \end{bmatrix}$$

T is <u>not</u> one-to-one because ${}_{\mathcal{C}}[T]_{\mathcal{B}}$ does not have a pivot in every column.

T <u>is</u> onto because ${}_{\mathcal{C}}[T]_{\mathcal{B}}$ has a pivot in every row.

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