MATH 54 SUMMER 2017, QUIZ 12

Are the following vectors in $\mathbb{P}_2$ linearly dependent? If not, explain why not. If so, find a nontrivial linear combination of them that is equal to the zero polynomial.

$$p = -2x^2 + 4x + 4$$
$$q = 3x^2 + 6x - 2$$
$$r = -2x^2 + x + 3$$

Let $\mathcal{B} = \{1, x, x^2\}$ be the usual basis for $\mathbb{P}_2$.

Then $[p]_{\mathcal{B}} = \begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}$ $[q]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 6 \\ 3 \end{bmatrix}$ $[r]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$.

So it suffices to check if $\begin{bmatrix} 4 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 3 \end{bmatrix}$, and $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ are linearly dependent.

$$\begin{bmatrix} 4 & -2 & 3 \\ 4 & 6 & 1 \\ -2 & 3 & -2 \end{bmatrix} \xrightarrow{R2 = R2 - R1} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 8 & -2 \\ -2 & 3 & -2 \end{bmatrix} \xrightarrow{R3 = R3 + \frac{1}{2}R1} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 8 & -2 \\ 0 & 2 & -1/2 \end{bmatrix}$$

$$\xrightarrow{R2 = \frac{1}{4}R2} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 2 & -1/2 \\ 0 & 2 & -1/2 \end{bmatrix} \xrightarrow{R3 = R3 - R2} \begin{bmatrix} 4 & -2 & 3 \\ 0 & 2 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\boxed{p, q, \text{ and } r \text{ are linearly dependent}}$ because the above matrix does not have a pivot in every column.

Solving the homogeneous equation:

x_3 is free
$$2x_2 - \tfrac{1}{2}x_3 = 0$$
$$4x_1 - 2x_2 + 3x_3 = 0$$

let $x_3 = 4$

$\Rightarrow$ $x_3 = 4$, $x_2 = 1$, $x_1 = -5/2$

So $\boxed{-5/2\, p + q + 4r = 0}$

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Check:

$$-\frac{5}{2}(-2x^2 + 4x + 4) + (3x^2 + 6x - 2) + 4(-2x^3 + x + 3)$$
$$= (5 + 3 - 8)x^2 + (-10 + 6 + 4)x + (-10 - 2 + 12) = 0$$