MATH 54 SUMMER 2017, QUIZ 11

Mark each of the following true or false. You do not have to provide an explanation.

(a) The set of invertible 3×3 matrices is a subspace of the vector space of 3×3 matrices.

False. The zero matrix, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is not invertible.

(b) The set of constant functions from $\mathbb{R}$ to $\mathbb{R}$ is a subspace of the vector space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$ (i.e. $C(\mathbb{R})$). (A function $f \colon \mathbb{R} \to \mathbb{R}$ is constant if for all real numbers x and y, $f(x) = f(y)$.)

True. ① the zero function (i.e. $\begin{matrix} \mathbb{R} \to \mathbb{R} \\ x \mapsto 0 \end{matrix}$) is constant.

② If f and g are constant then for all $x, y \in \mathbb{R}$, $(f+g)(x) = f(x) + g(x) = f(y) + g(y) = (f+g)(y)$. So $f+g$ is constant.

③ If f is constant and $c \in \mathbb{R}$ then for all $x, y \in \mathbb{R}$, $(cf)(x) = cf(x) = cf(y) = (cf)(y)$ So cf is constant.

(c) The set of polynomials with integer coefficients of degree at most 3 is a subspace of the vector space of polynomials with real coefficients of degree at most 3 (i.e. $\mathbb{P}_3$).

False. It is not closed under scalar multiplication. For instance $x^3 + x$ has integer coefficients, but $\frac{1}{2}(x^3 + x) = \frac{1}{2}x^3 + \frac{1}{2}x$ does not.

(d) The following vectors in $M_{2 \times 2}$ span all of $M_{2 \times 2}$ (recall that $M_{2 \times 2}$ is the vector space of all 2×2 matrices).

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

True. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2×2 matrix then

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + d\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

so $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \operatorname{span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(e) The following vectors in $\mathbb{P}_4$ are linearly independent: $x + 1$, $x^4 + x$, and $x^4 - 1$.

False.

$$(x^4 + x) - 1 \cdot (x + 1) - 1 \cdot (x^4 - 1) = 0$$

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