

MATH 54 SUMMER 2017, QUIZ 10

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 3 & 0 & 6 \\ 4 & 5 & 13 \end{bmatrix}$$

(a) Find a basis for Col A.

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 3 & 0 & 6 \\ 4 & 5 & 13 \end{bmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 3 & 0 & 6 \\ 4 & 5 & 13 \end{bmatrix} \xrightarrow{R_3=R_3-3R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & -6 & -6 \\ 4 & 5 & 13 \end{bmatrix} \xrightarrow{R_4=R_4-4R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & -6 & -6 \\ 0 & -3 & -3 \end{bmatrix} \\ & \xrightarrow{R_2 = -\frac{1}{5}R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & -6 & -6 \\ 0 & -3 & -3 \end{bmatrix} \xrightarrow{R_3=R_3+6R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & -3 & -3 \end{bmatrix} \xrightarrow{R_4=R_4+3R_2} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

The first two columns are pivot columns so a basis for

Col(A) is $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 5 \end{bmatrix} \right\}$

(b) Find a basis for Null A.

Solutions to $A\vec{x} = \vec{0}$:

x_3 is free

$$x_2 + x_3 = 0$$

$$x_1 + 2x_2 + 4x_3 = 0$$

$\Rightarrow x_3$ is free

$$x_2 = -x_3$$

$$x_1 = -2x_2 - 4x_3 = -2x_3$$

in parametric form:

$$x_3 \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$

so a basis for Null(A)

is $\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\}$