Review

1. For what values of c is the linear transformation given below one-to-one? For what values of c is it onto?

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 + 5x_3 \\ 2x_1 + 4x_3 \\ 3x_1 + 6x_3 \\ x_1 + x_2 + cx_3 \end{bmatrix}$$

2. For what values of c is the solution are the following vectors linearly independent?

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 5 \\ 4 \\ 6 \\ c \end{bmatrix}$$

3. For what values of c does the following matrix equation have a unique solution?

$$\begin{bmatrix} 1 & 1 & 5 \\ 2 & 0 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & c \end{bmatrix} \mathbf{x} = \mathbf{0}$$

Inverses

1. List as many conditions that are equivalent to "the $n \times n$ matrix A is invertible" as you can think of.

2. Find a matrix D such that $AD = I_2$ or explain why no such matrix exists.

$$A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 3 \end{bmatrix}$$

3. What is I_n^{-1}?

4. Suppose that A and B are invertible $n \times n$ matrices. Is AB always invertible? If not, give a counterexample. If so, what is the inverse of AB in terms of A, B, A^{-1}, and/or B^{-1}?

5. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

   (a) Is T one-to-one and onto?

   (b) What is $T^{-1}(\mathbf{e}_1)$? What is $T^{-1}(\mathbf{e}_2)$?

   (c) What is the standard matrix of T^{-1}?

6. Is A invertible? If so, find the inverse. If not, explain why not.

$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 10 \end{bmatrix}$$

7. With A the same as in the previous problem, solve the following equation:

$$A\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

8. Find a 3×3 matrix E such that multiplying by E is equivalent to the row operation "$R2 = R2 - 5R1$".