## Review

1. Suppose $A$ is a $3 \times 3$ matrix such that

$$A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}.$$

Find a solution to

$$A\mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}.$$

2. Let $\mathbb{P}$ be the set of all polynomials with real coefficients. Let $f\colon \mathbb{P} \to \mathbb{R}$ be the function defined by $f(p) = p(0)$. Is $f$ one-to-one? Is $f$ onto?

## Linear Transformations

1. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ be the function defined by $T(x, y) = (2x + y, x - y, y)$.
   (a) Find a matrix $A$ such that for all $\mathbf{x} \in \mathbb{R}^2$, $T(\mathbf{x}) = A\mathbf{x}$.
   (b) Is $T$ one-to-one?
   (c) Is $T$ onto?
2. Let $T$ be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^4$. Can $T$ be onto? If so, give an example. If not, explain why not.
3. Let $T$ be a linear transformation from $\mathbb{R}^9$ to $\mathbb{R}^4$. Can $T$ be one-to-one? If so, give an example. If not, explain why not.
4. True or false: If $T\colon \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation and $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are vectors in $\mathbb{R}^m$ whose span is $\mathbb{R}^m$ then $T(\mathbf{v}_1), \ldots, T(\mathbf{v}_p)$ span all of $\mathbb{R}^n$. If true, explain why. If not, provide a counterexample and provide an extra condition on $T$ that makes the statement true.

## Geometry of Solution Sets

1. Suppose $A$ is a $3 \times 3$ matrix such that the set of solutions to $A\mathbf{x} = \mathbf{0}$ is equal to $\text{span}\{\mathbf{a}, \mathbf{b}\}$ and $A\mathbf{c} = \mathbf{d}$. Find the set of solutions to $A\mathbf{x} = \mathbf{d}$.

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

2. **Challenge:** Try to figure out what the matrix $A$ is in the previous problem.

3. Suppose $A$ is a $3 \times 4$ matrix that is row equivalent to

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Find all solutions to

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$