The Heat Equation

1. Let $f(x,t) = e^{xt}\sin(x) + t^2 + 5 + \cos(x)$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial t}$ and $\frac{\partial^2 f}{\partial t^2}$.
2. Write down the definition of eigenvector of a linear transformation and then write down what it means for a function $z(x)$ to be an eigenvector with eigenvalue λ of the linear transformation $\frac{d^2}{dx^2}$.
3. Find all eigenvectors of $\frac{d^2}{dx^2}$ with eigenvalue -9.
4. Find all eigenvectors of $\frac{d^2}{dx^2}$ with eigenvalue 0.
5. Find a solution to the following differential equation.

$$\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}; \quad u(0,t) = u(0,L) = 0; \quad u(x,0) = \sin(\pi x/L) + 13\sin(5\pi x/L)$$

6. Find solutions to the heat equation with the boundary values given below. Try to use the same method we used earlier in class.

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(L,t)}{\partial x} = 0.$$

Definitions and Theorems

Definitions:

- Partial derivative
- PDE (partial differential equation)
- Heat Equation

Theorems:

- How to find solutions to the heat equation (which corresponds to finding eigenvectors of the second derivative).

Most important idea today: Finding solutions to linear PDEs like the heat equation uses exactly the same ideas we used to solve systems of linear ODEs: it's all about finding eigenvectors of some linear transformation.