

Differential Equations

1. Which of the following functions are solutions to the differential equation $y'' - y = 2 - t^2$?

(a) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(t) = t^2$

(c) $h: \mathbb{R} \rightarrow \mathbb{R}$ defined by $h(t) = t^2 + e^t$

(b) $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $g(t) = te^t$

2. Which of the functions in problem 1 are solutions to the initial value problem

$$y'' - y = 2 - t^2$$

$$y(0) = 1$$

$$y'(0) = 1$$

3. Find the general solution of the following differential equations.

(a) $y'' - 2y' - 3y = 0$

(c) $y''' + 5y'' + 4y' = 0$

(b) $y'' - 5y' = 0$

4. Find a differential equation for which $e^{7t} + 4e^{-3t}$ is a solution.

5. Find the solution to each of the following initial value problems.

(a) $y'' + y' = 0, y(0) = 2, y'(0) = 1$

(b) $y''' + 5y'' + 4y' = 0, y(0) = 8, y'(0) = -9, y''(0) = 33$

Definitions and Theorems

Definitions:

- Differential Equation, ODE
- Solution to a differential equation
- Initial value problem (IVP)
- Linear, constant coefficient, homogeneous ODE
- Auxiliary equation

Most important idea today: Finding the general solution to a linear, constant coefficient, homogeneous ODE just means finding the kernel of some linear transformation.