Review Questions

1. How many solutions does a system of linear equations have if the coefficient matrix in REF has:
   (a) A pivot in every row?
   (b) A pivot in every column?
   (c) A free variable (i.e. a column with no pivot)?
   (d) More columns than rows?
   (e) More rows than columns?

2. For what values of c are the following augmented matrices consistent?

   (a) $$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & c \end{array}\right]$$

   (b) $$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ c & 3 & -2 \\ 0 & 0 & 0 \end{array}\right]$$

Span

1. Is it possible to add together multiples of $\mathbf{a}$ and $\mathbf{b}$ to get $\mathbf{c}$?

   (a) $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\mathbf{c} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

   (b) $\mathbf{a} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $\mathbf{c} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

2. Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\}$?

3. (a) What is span $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$?
   (b) What is span $\left\{ \begin{bmatrix} 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$?

4. If $\mathbf{u}$ and $\mathbf{v}$ are both in span$\{\mathbf{w}_1, \mathbf{w}_2\}$ then is $5\mathbf{u} - 2\mathbf{v}$ also in the span of $\mathbf{w}_1$ and $\mathbf{w}_2$?

5. (a) Is it possible to find two vectors in $\mathbb{R}^2$ that *don't* span all of $\mathbb{R}^2$?
   (b) Is is possible to find two vectors in $\mathbb{R}^2$ whose span does not include $\mathbf{0}$?
   (c) Is it possible to find two vectors in $\mathbb{R}^3$ whose span is all of $\mathbb{R}^3$?

6. Draw span $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$ and span $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$.

Linear Independence

1. Show that if $\mathbf{v}_1$ and $\mathbf{v}_2$ are nonzero vectors in $\mathbb{R}^n$ that are linearly dependent then $\mathbf{v}_1 = a\mathbf{v}_2$ for some real number a.

2. Show that if $\mathbf{0} \in \{\mathbf{v}_1, \ldots, \mathbf{v}_m\}$ then $\mathbf{v}_1, \ldots, \mathbf{v}_m$ are linearly dependent.

3. Are the following vectors linearly independent?

$$\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

4. If $m > n$, is it possible to find $\mathbf{v}_1, \ldots, \mathbf{v}_m$ in $\mathbb{R}^n$ that are linearly independent?