After finishing the quiz, work on the first four questions.

Diagonalization

1. What is A^{100}?
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

2. If $A = PBP^{-1}$ then what is A^{100}?

3. (a) Suppose $\mathbf{v}_1$ and $\mathbf{v}_2$ are eigenvectors of a matrix A with corresponding eigenvalues 5 and -3. What is $A^{100}\mathbf{v}_1$? What about $A^{100}\mathbf{v}_2$?
   (b) If $\mathbf{v}_3 = 2\mathbf{v}_1 + 6\mathbf{v}_2$, what is $A^{100}\mathbf{v}_3$?

4. $$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix} \quad \mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$
   (a) What is $[\mathbf{e}_1]_{\mathcal{B}}$? What is $[\mathbf{e}_2]_{\mathcal{B}}$?
   (b) What are $A^{100}\mathbf{e}_1$ and $A^{100}\mathbf{e}_2$? (Hint: the vectors in $\mathcal{B}$ are eigenvectors of A.)
   (c) What is A^{100}? (Hint: Use the previous part.)

5. What is the maximum number of eigenvalues a 5×5 matrix can have?

6. Try to diagonalize the following matrices (not all of them are necessarily diagonalizable.)
   (a) $\begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ -3 & -6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$

7. Find a matrix A such that $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are eigenvectors of A with corresponding eigenvalues -5 and 1.

8. For each statement below, explain why it is true or provide a counterexample to show it is false.
   (a) Every invertible matrix is diagonalizable.
   (b) Every diagonalizable matrix is invertible.
   (c) If A is a nonzero matrix and $A^2 = 0$ then A is not diagonalizable.
   (d) Every 2×2 matrix with more than one eigenvalue is diagonalizable.
   (e) Every upper triangular matrix is diagonalizable.