Review

1. Suppose a, b, and c are real numbers and that a is nonzero. For what values of d is the following matrix not invertible?

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Determinant

1. Find the determinant of both matrices below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -6 \\ 1 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 & 4 & 7 \\ 1 & 1 & 0 & 5 & 0 \\ 4 & 5 & 0 & 2 & 2 \\ 0 & 0 & 0 & 15 & 0 \\ 1 & 1 & 0 & -1 & 0 \end{bmatrix}$$

2. Using the formula for the determinant, how many times do you have to multiply two numbers when finding the determinant of a 3×3 matrix when all its entries are not zero? What about a 5×5 matrix?

3. What is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 5 & -7 & 8 \\ 0 & 2 & 10 & 9 \\ 0 & 0 & 3 & 21 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

4. What is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ -1 & -2 & -3 & -5 \\ 5 & 10 & 20 & 20 \end{bmatrix}$$

5. (a) If A is an invertible matrix, what is $\det(A^{-1})$?
   (b) What is $\det(5I_n)$?
   (c) If A and B are similar matrices, can you say anything about $\det(A)$ and $\det(B)$?
   (d) True or false: $\det(A + B) = \det(A) + \det(B)$ for all $n \times n$ matrices A and B.

6. **Challenge Problem:** If $\alpha_1, \ldots, \alpha_n$ are real numbers, what is the determinant of the following matrix?

$$\begin{bmatrix} 1 & 1 & \ldots & 1 \\ \alpha_1 & \alpha_2 & \ldots & \alpha_n \\ \alpha_1^2 & \alpha_2^2 & \ldots & \alpha_n^2 \\ & & \vdots & \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \ldots & \alpha_n^{n-1} \end{bmatrix}$$