

Linear Equations and Row Reduction

1. Find all solutions of each system of linear equations below.

$$\begin{array}{lll} \text{(a)} & x_1 + 2x_2 = 7 & \text{(b)} & x_1 + 2x_2 = 3 & \text{(c)} & x_1 + 2x_2 = 3 \\ & 3x_1 + 9x_2 = 24 & & 3x_1 + 6x_2 = 9 & & 3x_1 + 6x_2 = 10 \end{array}$$

2. Write each of the following systems as augmented matrices.

$$\begin{array}{lll} \text{(a)} & x_1 + 2x_2 = 7 & \text{(b)} & x_1 + 2x_2 = 3 & \text{(c)} & x_1 + 2x_2 = 3 \\ & 3x_1 + 9x_2 = 24 & & 3x_1 - 6x_2 = 9 & & 3x_1 - 3x_3 = 10 \\ & & & 5x_1 = -9 & & \end{array}$$

3. For each augmented matrix, find all solutions to the system of linear equations that it represents.

$$\begin{array}{lll} \text{(a)} & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array} \right] & \text{(b)} & \left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -7 \end{array} \right] & \text{(c)} & \left[\begin{array}{ccc|c} 1 & 3 & 6 & 1 \\ 0 & 2 & 1 & 7 \\ 0 & 0 & 3 & 9 \end{array} \right] \end{array}$$

4. Label each of the following **coefficient** matrices as “REF,” “RREF,” or “neither.” For each matrix in REF or RREF, circle the pivots and box the columns that correspond to free variables.

$$\begin{array}{lll} \text{(a)} & \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} & \text{(c)} & \begin{bmatrix} 1 & 2 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} & \text{(e)} & \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ \text{(b)} & \begin{bmatrix} 2 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \text{(d)} & \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 5 \end{bmatrix} & \text{(f)} & \begin{bmatrix} 0 & 0 & 2 & -5 & 7 \\ 0 & 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \end{array}$$

5. Use row reduction to find solutions to each of the following systems of linear equations.

$$\begin{array}{ll} \text{(a)} & \begin{array}{l} 3x_1 + 6x_2 + 3x_3 = -3 \\ 5x_1 - 3x_2 + 18x_3 = 8 \\ 7x_1 + 2x_2 + 19x_3 = 5 \end{array} & \text{(b)} & \begin{array}{l} x_1 + 2x_2 = 3 \\ 3x_1 - 6x_2 = 9 \\ x_1 + x_2 = 10 \end{array} \end{array}$$

6. For what values of h is the following system consistent?

$$\begin{array}{l} x_1 + hx_2 = 1 \\ \quad \quad 2x_2 = 2 \\ 3x_1 - x_3 = 3 \end{array}$$