## Functions

1. Suppose $f\colon \mathbb{N} \to \mathbb{N}$ is the function defined by $f(n) = 2n$ and $g\colon \mathbb{N} \to \{0, 1\}$ is the function defined by
$$g(n) = \begin{cases} 0 & \text{if } n \text{ is even} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$$
   (a) What is $(g \circ f)(5)$?
   (b) What is the range of $g \circ f$?
   (c) Is $g \circ f$ one-to-one? Onto?
2. Check whether each function is invertible. If it is invertible, find its inverse.
   (a) The function $f\colon \{1, 2, 3\} \to \{4, 5, 6\}$ defined by
$$f(n) = \begin{cases} 5 & \text{if } n = 1 \\ 6 & \text{if } n = 2 \\ 4 & \text{if } n = 3. \end{cases}$$
   (b) The function $g\colon \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2$.
   (c) The function $h\colon \mathbb{R} \to \mathbb{R}$ defined by $h(x) = 2x - 3$.

## Linear transformations and Matrices

1. For each pair of linear transformations $S$ and $T$ below, find the standard matrix of $S \circ T$.
   (a) $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is rotation by 90° counterclockwise and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is expansion by 3 in the horizontal direction.
   (b) $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is expansion by 3 in the horizontal direction and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is rotation by 90° counterclockwise.
   (c) $S\colon \mathbb{R} \to \mathbb{R}^3$ defined by
$$S(x) = \begin{bmatrix} x \\ x \\ x \end{bmatrix}$$
   and $T\colon \mathbb{R}^3 \to \mathbb{R}$ defined by
$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x + y + z.$$
   (d) $S\colon \mathbb{R}^2 \to \mathbb{R}^4$ has standard matrix
$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 0 \\ 4 & 1 \end{bmatrix}$$
   and $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is rotation by 90° counterclockwise.
2. For each item in the previous question, check whether $S \circ T$ is invertible. If it is, find the standard matrix of its inverse.