Review

Consider the following system of equations.

$$\begin{aligned} x_1 &- hx_2 &&= 1 \\ x_1 &- x_2 &&= 0 \end{aligned}$$

For which values of h is the system consistent? Explain what is going on geometrically.

Geometry of Solution Sets

1. Suppose A is a 3×3 matrix such that the set of solutions to $A\mathbf{x} = \mathbf{0}$ is equal to $\text{span}\{\mathbf{a}, \mathbf{b}\}$ and $A\mathbf{c} = \mathbf{d}$. Find the set of solutions to $A\mathbf{x} = \mathbf{d}$.

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ 9 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}$$

2. Suppose A is a 3×4 matrix that is row equivalent to

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and such that

$$A \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Find all solutions to

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$