Similar Matrices

1. Suppose A and B are similar 2×2 matrices and $\det(A) = 5$. What can you say about $\det(B)$?
2. Suppose A is a 2×2 matrix which is similar to the 0 matrix (i.e. the 2×2 matrix whose entries are all 0). What can you say about A?
3. Suppose A is a 2×2 matrix which is similar to I_2. What can you say about A?

Diagonalization

1. Try to diagonalize the following two matrices.

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$$

2. Find a 2×2 matrix A such that $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ is an eigenvector of A with eigenvalue 5 and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of A with eigenvalue -1.

Why Diagonalize?

1. Suppose $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} A \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1}$.
   (a) What is A^{100}?
   (b) What is B^{100}?
2. Find $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}^{2021}$
3. **Challenge Problem:** (Repeated from a previous worksheet). What is $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}^{2021}$?

Extra Problems

1. What is the maximum number of eigenvalues a 5×5 matrix can have? What is the minimum number it can have and still be diagonalizable?
2. For each statement below, explain why it is true or provide a counterexample to show it is false.
   (a) Every 5×5 matrix with 5 distinct eigenvalues is diagonalizable.
   (b) Every invertible matrix is diagonalizable.
   (c) Every diagonalizable matrix is invertible.

(d) If A is a nonzero matrix and $A^2 = 0$ then A is not diagonalizable.

(e) Every 2×2 matrix with more than one eigenvalue is diagonalizable.

(f) Every upper triangular matrix is diagonalizable.