

Abstract Vector Spaces

- Which of the following are vector spaces?
 - The set of sequences of real numbers that converge to 0.
 - The set of sequences of real numbers that converge to 1.
 - The set of letters in the Latin alphabet.
 - The set of 2×3 matrices in RREF.
 - The set of differentiable functions on the real numbers.
- Are the polynomials $1, x^2, 3x^2 - 2$ linearly independent?
- Is the sequence $(1, 0, 1, 0, \dots)$ in the span of $(1, 1, 1, 1, \dots)$ and $(1, -1, 1, -1, \dots)$?
- Are the following 2×2 matrices linearly independent?

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

- Which of the following are linear transformations?
 - $T: \mathbb{P}_3 \rightarrow \mathbb{P}_3$ defined by $T(p) = \frac{dp}{dx}$.
 - $T: M_{2 \times 2} \rightarrow M_{2 \times 2}$ defined by $T(A) = A + I_2$.
 - $T: C([0, 1]) \rightarrow \mathbb{R}$ defined by $T(f) = \int_0^1 f(x) dx$.