

## MATH 54, FALL 2016, QUIZ 12

In each of the following problems, find a solution to the heat equation,  $\frac{\partial u}{\partial t} = \beta \frac{\partial^2 u}{\partial x^2}$ , with boundary values  $u(0, t) = u(L, t) = 0$  and initial conditions as given in each problem.

- (1) Find a solution to the heat equation with initial condition  $u(x, 0) = 3f(x)$  where  $f: [0, L] \rightarrow \mathbb{R}$  is a continuous function such that  $f(0) = f(L) = 0$  and

$$\int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{1}{n^2}.$$

- (2) Find a solution to the heat equation with initial condition

$$u(x, 0) = -5 \sin\left(\frac{32\pi x}{L}\right) + 13 \sin\left(\frac{307\pi x}{L}\right).$$

Hint:  $\int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = \frac{L}{2}$ .

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