Math 10B, Quiz 11 Solutions

1. (9 points) Use Gaussian elimination to reduce the augmented matrix below to one in which the coefficient matrix is upper triangular.

$$\left[\begin{array}{ccc|c} 2 & 4 & 6 & 1 \\ -2 & -2 & -1 & 0 \\ 3 & 12 & 16 & 0 \end{array}\right]$$

Solution:

$$\left[\begin{array}{ccc|c} 2 & 4 & 6 & 1 \\ -2 & -2 & -1 & 0 \\ 3 & 12 & 16 & 0 \end{array}\right] \xrightarrow{\frac{1}{2}R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ -2 & -2 & -1 & 0 \\ 3 & 12 & 16 & 0 \end{array}\right] \xrightarrow{R_2+2R_1 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ 0 & 2 & 5 & 1 \\ 3 & 12 & 16 & 0 \end{array}\right]$$

$$\xrightarrow{R_3-3R_1 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ 0 & 2 & 5 & 1 \\ 0 & 6 & 7 & -3/2 \end{array}\right] \xrightarrow{\frac{1}{2}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ 0 & 1 & 5/2 & 1/2 \\ 0 & 6 & 7 & -3/2 \end{array}\right]$$

$$\xrightarrow{R_3-6R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1/2 \\ 0 & 1 & 5/2 & 1/2 \\ 0 & 0 & -8 & -9/2 \end{array}\right]$$

2. (2 points) If A and B are the matrices shown below, then AB is defined.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & -4 \end{bmatrix}$$

√ **True** ◯ False

Solution: A is a 2×3 matrix and B is a 3×3 matrix so AB is defined.

3. (2 points) The matrix B in the previous question is invertible.

◯ True √ **False**

Solution: If you calculate the determinant using your favorite method, you will get 0 and so the matrix is not invertible.

4. (2 points) For the matrices C and D shown below, D is the inverse of C.

$$C = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix}$$

◯ True √ **False**

Solution: It is possible to solve this problem by actually finding the inverse of C using the algorithm from lecture. However, this is pretty annoying. Instead, recall that the definition of the inverse of the matrix C is the matrix that, when multiplied by C, gives the identity matrix. So to check if D is the inverse of C, we can just multiply C and D and see if we get the identity matrix. The product of C and D is

$$CD = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

Since the result is not the identity matrix (since it has tens on the diagonal instead of ones), D is not the inverse of C. In fact, the inverse of C is $\frac{1}{10}D$.