

## Dynamics Worksheet 1

- Suppose you roll two four-sided dice 100 times each and each time calculate the product of the two rolls. True or false: if you want to perform a  $\chi^2$  test to check if the dice are fair based on the data you recorded then the degrees of freedom is 3.
- For each of the following, verify whether the given formula is a solution to the given recurrence relation.

(a) Recurrence relation:

$$a_n = \sqrt{a_{n-1}^2 + 1}; a_0 = \sqrt{5}$$

Formula:

$$a_n = \sqrt{n + 5}$$

(b) Recurrence relation:

$$a_n = 2^{a_{n-1}} + a_{n-2}; a_0 = 1, a_1 = 2$$

Formula:

$$a_n = \sqrt{n + 5}$$

(c) Recurrence relation:

$$a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$$

Formula:

$$a_n = n^2 + 3n$$

(d) Recurrence relation:

$$a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$$

Formula:

$$a_n = 3^n$$

(e) Recurrence relation:

$$a_n = 2a_{n-1} + 3a_{n-2}; a_0 = 0, a_1 = 4$$

Formula:

$$a_n = 3^n - (-1)^n$$

- Find a formula for the  $n^{\text{th}}$  Fibonacci number. Recall that the first two Fibonacci numbers (i.e. the  $0^{\text{th}}$  and  $1^{\text{st}}$  Fibonacci numbers) are both 1 and that to get the next Fibonacci number, you add the previous two. First formulate this as a recurrence relation and then try to solve it.
- Find a solution to the following recurrence relation:

$$a_n = n \cdot a_{n-1}; a_0 = 1$$

- Challenge Question:** Look up the rules to the Tower of Hanoi game and find a formula for the least number of moves it takes to win the game when there are  $n$  disks.