MATH 10B, SPRING 2017, QUIZ 7

(1) Suppose you draw five cards from a standard deck of 52.

(a) What is the probability that you get exactly three red cards?

> Let Ω be all unordered sets of five distinct cards. Let A be the event that there are exactly three red cards. Since each outcome in Ω is equally likely, $P(A) = \frac{|A|}{|\Omega|}$. To pick a set of five cards with exactly three red cards, we need to choose 3 cards out of the 26 red cards and 2 black cards out of the 26 black cards. So
> $$|A| = \binom{26}{3}\binom{26}{2}.$$
> And since Ω is all ways to choose 5 cards from 52 where order doesn't matter,
> $$|\Omega| = \binom{52}{5}.$$
> Therefore
> $$P(A) = \frac{\binom{26}{3}\binom{26}{2}}{\binom{52}{5}}.$$
> This is an example of the hypergeometric distribution that was discussed in lecture.

(b) Now suppose that if you don't get exactly three red cards, you replace all five cards, reshuffle the deck, and try again. You keep doing this until you do get exactly three red cards. What is the probability that you have to repeat this exactly seven times?

> Let p be the answer to part (a)—i.e. the probability of success. In order for it to take exactly 7 attempts, we need to fail exactly six times and then succeed once. Since we reshuffle the cards, each attempt is independent. Thus we can multiply the probabilities to get $(1-p)^6 p$. This is an example of the geometric distribution that was discussed in lecture.

(2) Suppose X and Y are independent random variables such that $E[X] = 2$, $E[Y] = -3$, $\mathrm{Var}[X] = 1$, and $\mathrm{Var}[Y] = 4$. Find each of the following:

- $E[X^2] = 5$

> Since $\mathrm{Var}[X] = E[X^2] - E[X]^2$, $1 = E[X^2] - 2^2$.

- $E[X + Y] = -1$

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By linearity of expectation, $E[X+Y] = E[X] + E[Y]$.

- $E[XY] = -6$

Since X and Y are independent, $E[XY] = E[X]E[Y]$.

- $\text{Cov}[X, Y] = 0$

Since X and Y are independent.

- $\text{Var}[X+Y] = 5$

Since X and Y are independent, $\text{Var}[X+Y] = \text{Var}[X] + \text{Var}[Y]$.