MATH 10B, SPRING 2017, QUIZ 6

(1) Suppose you are playing a game where someone rolls two fair 6-sided dice. If both rolls are ones, you win a million dollars.

(a) If you are told that the first roll is a one, what is the chance that you will win?

> Let A be the event that the first roll is a one and B the event that both rolls are one. The question is asking for $P(B \mid A)$. By definition,
> $$P(B \mid A) = \frac{P(B \cap A)}{P(A)}.$$
> The intersection of A and B is just B—if both rolls are one then the first roll must be one. The probability that both rolls are one is
> $$P(B) = \frac{1}{6^2}$$
> and the probability that the first roll is one is
> $$P(A) = \frac{1}{6}.$$
> Therefore the answer is
> $$\frac{1/6^2}{1/6} = \frac{1}{6}.$$

(b) If you are told that at least one of the rolls is a one, what is the chance that you will win?

> Let C be the event that at least one roll is a one and let B be as in the solution to part (a). The question is asking for $P(B \mid C)$. Once again by definition of conditional probability,
> $$P(B \mid C) = \frac{P(B \cap C)}{P(C)}.$$
> As before, $B \cap C = B$ since if both rolls are one, then at least one roll is one. So we just need to calculate $P(C)$. To do this, let's define our sample space Ω to be the set of all pairs of numbers between one and six. Since the dice are fair, every outcome in Ω is equally likely. And $|\Omega| = 36$. To find $|C|$, observe that the number of pairs of numbers that don't contain a one is $5 \times 5 = 25$. So $|C| = |\Omega| - 25 = 11$. Therefore
> $$P(C) = \frac{11}{36}$$

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and so

$$P(B \mid C) = \frac{1/6^2}{11/36} = \frac{1}{11}.$$

(2) Suppose that you roll two fair 6-sided dice. Let A be the event that both rolls are the same. Let B be the event that at least one of the rolls is a one. Are A and B independent? Use the definition of independence to justify your answer.

The definition of independence is that A and B are independent if and only if $P(A \cap B) = P(A)P(B)$. We already found in question 1, part (b), that $P(B) = 11/36$. We can also calculate that $P(A) = 6/36 = 1/6$ since there are exactly six outcomes in the sample space defined above in which both rolls are the same. And the intersection of A and B is just the event that both rolls are one, which we already found to have probability $1/36$. Therefore we have

$$\begin{aligned} P(A \cap B) &= \frac{1}{36} \\ &\neq \frac{1}{6} \cdot \frac{11}{36} \\ &= P(A)P(B). \end{aligned}$$

Therefore the events are not independent.