

MATH 10B, SPRING 2017, QUIZ 4

For all questions below, a numerical answer is not required. It is sufficient to write an expression using functions that we defined in class. Make sure to explain your answers.

- (1) (a) If there are 100 hours of work to do and you have 6 employees, how many ways are there to divide the work between the 6 employees? Assume that employees cannot work fractions of hours—e.g. an employee cannot work for 5.4 hours.

In terms of the twelvefold way, this corresponds to 100 indistinguishable balls and 6 distinguishable boxes. So we can solve it using stars and bars, giving us

$$\binom{100 + 6 - 1}{100}.$$

- (b) What if every employee must work at least 10 hours?

This is just like part (a), but we first have to put 10 “balls” in every box, leaving 40 indistinguishable balls and 6 boxes. At this point, using stars and bars gives us

$$\binom{40 + 6 - 1}{40}.$$

- (2) (a) How many ways are there for 20 students to form 6 study groups? Assume every student must be in exactly one study group and no study group can be empty.

In terms of the twelvefold way, this corresponds to 20 distinguishable balls and 6 indistinguishable boxes, all of which must be nonempty. By definition, this is equal to

$$S(20, 6).$$

- (b) How many ways are there for the 20 students to form study groups if they have to form at most 6 study groups, but they are allowed to form fewer than 6? Assume as before that every student must be in exactly one study group.

This is just like the last problem, but the boxes are no longer required to be nonempty. So as we saw in class, this is

$$\sum_{k=1}^6 S(20, k).$$