

MATH 10B, SPRING 2017, QUIZ 1 SOLUTIONS

- (1) How many 4 digit numbers are there which contain only even digits?

There are 5 even digits in total: 0,2,4,6, and 8. To choose a 4 digit number with only even digits, we have to choose a digit four times. But the first digit should not be a 0 since that would not give us a 4 digit number. So we have 4 choices for the first digit and 5 choices for the three other digits, giving $4 \cdot 5^3$ total numbers.

- (2) Suppose you have 10 colors of paint. You can also combine these colors to form new colors. Each combination of the ten original colors gives a different color of paint, but proportions don't matter (so a little green mixed with a lot of red gives the same color as a little red mixed with a lot of green). How many colors can you create, including the original ten colors? (By the way, for the purposes of this problem the absence of paint does not count as a color.)

Because proportions and order of mixing colors don't matter, making a new color simply consists of choosing some of the ten paints to mix together. In other words, the colors we can create correspond to subsets of the original 10 paint colors. There are 2^{10} total subsets because for each of the 10 colors we can choose to include it in the subset or not to include it, giving us 10 choices with two possibilities for each. However, the empty subset (the subset that doesn't contain any of the 10 paint colors) does not correspond to a color we can create. So the total number of colors we can create is $2^{10} - 1$.