MATH 10B, SPRING 2017, QUIZ 10

Solve the following differential equations.

(1) $y'' + 7y' + 10y = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 1$.

This is a linear, homogeneous equation with constant coefficients. So we can use the characteristic polynomial method. The characteristic polynomial is

$$\lambda^2 + 7\lambda + 10 = (\lambda + 2)(\lambda + 5).$$

The roots are -2 and -5 so the general solution to the differential equation is

$$y(t) = C_1 e^{-2t} + C_2 e^{-5t}.$$

Using the initial conditions to solve for C_1 and C_2, we find that

$$1 = y(0) = C_1 e^{-2\cdot 0} + C_2 e^{-5\cdot 0} = C_1 + C_2$$
$$1 = y'(0) = -2C_1 e^{-2\cdot 0} - 5C_2 e^{-5\cdot 0} = -2C_1 - 5C_2$$

Solving this system of linear equations gives us $C_1 = 2$ and $C_2 = -1$. Therefore the final solution is

$$y(t) = 2e^{-2t} - e^{-5t}.$$

(2) $ty' - 4y = t^2$

For this equation, we can use the integrating factor method. First we divide by t to isolate y'. This gives us

$$y' - \frac{4}{t} y = t.$$

The integrating factor is

$$e^{\int -4/t\, dt} = e^{-4\ln|t|} = \left(e^{\ln|t|}\right)^{-4} = |t|^{-4} = t^{-4}.$$

Therefore

$$t^{-4} y(t) = \int t^{-4} t\, dt = \int t^{-3}\, dt = -\frac{1}{2t^2} + C.$$

Solving for $y(t)$ we get a final solution of

$$y(t) = -\frac{t^2}{2} + Ct^4.$$

(3) $t^2 y' = -y^2$

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This equation is separable. So we have

$$\int -\frac{1}{y^2}\,dy = \int \frac{1}{t^2}\,dt.$$

Therefore

$$\frac{1}{y} = -\frac{1}{t} + C.$$

Solving for $y(t)$ gives us a final solution of

$$y(t) = \frac{1}{-\frac{1}{t} + C}.$$

By the way, this is *not* equal to $-t + C$ or to $-t + \frac{1}{C}$.