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Discrete breathers and intrinsic energy localization in one-dimensional diatomic granular crystals

S. Job[a], N. Boechler[b], G. Theocharis[b], P.G. Kevrekidis[c], M.A. Porter[d] and C. Daraio[b]

[a]Laboratoire d'Ingénierie des Systèmes Mécaniques et des MAtériaux, Supméca - 3, rue Fernand Hainaut - 93407 St Ouen Cedex
[b]Graduate Aerospace Laboratories, California Institute of Technology, Pasadena, 91125, USA
[c]Department of Mathematics and Statistics, University of Massachusetts, Amherst, 01003-4515, USA
[d]Mathematical Institute, University of Oxford, OX1 3LB Oxford, UK
stephane.job@supmeca.fr

We report the experimental observation of modulational instability and discrete breathers in a one-dimensional diatomic granular crystal composed of elastic spheres that interact via the nonlinear Hertz potential. Our crystal consists of an alignment of 19.05-mm-diameter steel and aluminium spheres compressed by a static load in the axial direction. We first characterize the linear spectrum of the crystal by analyzing the low amplitudes transmitted vibrations; we observe the existence of acoustic and optical bands separated by a band gap. We then illustrate theoretically and numerically the modulational instability of the lower edge of the optical band. We finally show experimentally that modulational instability leads to the dynamical formation of long-lived and spatially localized breather structures [Phys. Rev. Lett. 104, 244302 (2010); Phys. Rev. E 82, 056604 (2010)].

1 Introduction

Discrete breathers (DB) and intrinsic localized modes (ILM) have been a central theme in nonlinear investigations during the past two decades [1,2]. Their original theoretical proposal in settings such as anharmonic lattices [3] and the rigorous proof of their existence under fairly general conditions [4] motivated studies of such modes in a diverse host of applications [5]. Granular crystals, in turn, consist of closely packed assemblies of elastically interacting particles. Recent interest has arisen from their tunable dynamic response encompassing linear, weakly nonlinear, and strongly nonlinear regimes [6,7]. Such flexibility, arising from the nonlinear contact interaction between particles, makes them ideal not only as toy models for probing the physics of granular materials but also for the implementation of engineering applications, including shock and energy absorbing layers [8], actuating devices [9], and sound scramblers [10]. Only recently have nonlinear localized modes begun to be explored in granular crystals. Previous studies have focused on metastable breathers in an acoustic vacuum [11], the observation of localized oscillations near a defect [12,13], and one-dimensional diatomic crystals restricted to linear dynamics due to welded sphere contacts [14]. Understanding and controlling energy localization in granular crystals might lead to new energy-harvesting or filtering devices.

We report recent achievements in this proceedings article, concerning the experimental observation of intrinsic energy localization and the existence of discrete breathers in a one-dimensional diatomic granular crystal [15]. We also report a systematic and rigorous study of the existence and stability of DB in such systems [16].

2 Experimental setup

The experimental setup shown in Fig. (1) is a one-dimensional diatomic granular crystal. It is assembled by alternating aluminum spheres (radius $R_a = 9.525$ mm, mass $m_a = 9.75$ g, Young modulus $E_a = 73.5$ GPa and Poisson ratio $\upsilon_a = 0.33$) and stainless steel spheres ($R_b = R_a$, $m_b = 28.84$ g, $E_b = 193$ GPa, $\upsilon_b = 0.30$). The spheres are held in place using four polycarbonate restraining cylinders. At one end of the crystal, a pre-compressive force is applied using a lever-mass system. The granular crystal is dynamically driven with a piezoelectric actuator fixed on a steel plate at the other end. The evolution of the force versus time, as the vibrations propagate in the alignment, is visualized using periodically placed thin piezoelectric force sensors inserted inside selected particles (preserving the inertia and the bulk stiffness of the original bead [7,10]). The static load is measured using a calibrated strain gauge cell placed between the lever arm and the last bead of the crystal.

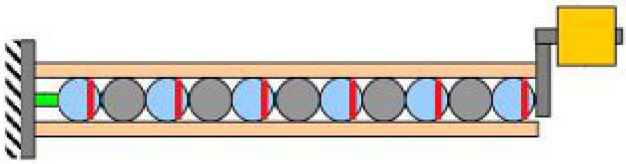

Figure 1: A one-dimensional diatomic granular crystal between a piezoelectric actuator (left) and a lever-mass system (left). The crystal is a pre-compressed alignment of aluminum (blue) and steel (gray) spheres. Thin force sensors (red) are inserted inside few of the spheres.

3 Theoretical model and simulations

We model [15,16] a one-dimensional diatomic crystal of N spheres as a chain of nonlinear oscillators [6]:

$$m_n \ddot{u}_n = A\left[\delta_0 + u_{n-1} - u_n\right]_+^p - A\left[\delta_0 + u_n - u_{n+1}\right]_+^p , \qquad (1)$$

where $\left[Y\right]_+$ denotes the positive part of Y, δ_0 is the overlap of spheres under static load F_0, u_n is the displacement of the *n*th sphere around the static equilibrium, the masses are $m_{odd} = m_a$ and $m_{even} = m_b$, and the coefficient A depend on the exponent p and the geometry/material properties of adjacent beads [15,16]. The exponent $p = 3/2$ yields the Hertz potential law between adjacent spheres [17].

From Eq. (1), we compute the linear dispersion relation of our system from the linearization of Eq. (1). For diatomic crystals, this curve contains two branches (acoustic and optical). At the edge of the first Brillouin zone, the linear spectrum possesses a gap between the upper cutoff frequency of the acoustic branch and the lower cutoff frequency of the optical branch.

Within weakly nonlinear perturbation, $\left(u_n - u_{n-1}\right) \ll \delta_0$, and with $p = 3/2$, one can approximates Eq. (1) in a power series expansion of the forces, up to quartic displacement terms, to yield the following expression:

$$m_n \ddot{u}_n = \sum_{k=2}^{4} K_k \left[\left(u_{n+1} - u_n\right)^{k-1} - \left(u_n - u_{n-1}\right)^{k-1}\right]. \qquad (2)$$

Eq. (2) constitutes a diatomic variant of the Fermi-Pasta-Ulam (FPU) nonlinear oscillator chain [18]. Such an equation is known to have a solution in the form of an asymmetric localized mode, induced by nonlinearity, inside the gap of the linear spectrum (i.e., a gap soliton) [19]. This arises from the modulational instability (MI) [20] of the optical lower cutoff phonon mode.

We present, in Fig. 2, an example of a simulated DB. The simulation is achieved by imposing a harmonic displacement to the first sphere of the alignment, at the lower optical cutoff frequency. One clearly sees the formation of a localized oscillation inside the crystal. The plot of the force versus time at a location where the DB develops reveals an exponential increase, characteristic of the MI.

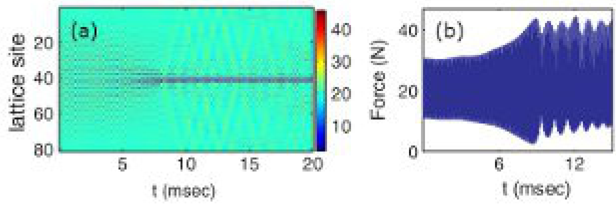

Figure 2: Numerical simulation (from [15]) of a DB near the lower optical cutoff frequency. (a) Spatiotemporal evolution of the force. (b) Force versus time for particle 40.

In [16], we presented systematic computations of the intrinsically localized excitations that 1D diatomic granular crystals can support. In particular, we examined two families of discrete gap breather solutions, in the gap of the linear spectrum between the acoustic and optical bands. One of them consists of heavy-symmetric DB and the other one consists of light-asymmetric DB. We found that the heavy-symmetric DB branch is always unstable. In turn, the light-asymmetric DB have the potential to be stable as long as their frequency lies sufficiently close to the optical band.

2 Experiments in the linear and weakly nonlinear regimes

The linear spectrum of our diatomic crystal is experimentally characterized by applying a low-amplitude broadband frequency uniform electrical noise to the piezoelectric actuator. The dynamic force is measured from the signal of a thin piezoelectric force sensor inserted inside a particle of the alignment. The transfer function is obtained as the ratio of the power spectral density (PSD) of the force normalized to the driving voltage. The transfer function is shown in Fig. 3. This spectrum clearly shows forbidden bands (i.e., gaps) and two pass bands bounded by cutoff frequencies. These frequencies were shown to be in fairly satisfactory agreement with the model and the numerical simulation exposed in the previous section [15,16].

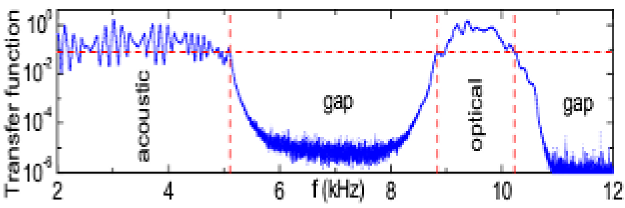

Figure 3: Experimental transfer function (from [15]) of the vibrations measured within the linear regime in a diatomic granular crystal made of 81 beads and a 20N static force.

The 81-bead diatomic crystal is then driven with a high-amplitude 30 ms long sine voltage at the lower optical cutoff frequency. Force sensors are placed in particles 2, 4, 7, 12, and 14. Forces measurements shown in Fig. (4) reveal the presence of a DB, whose frequency differs from the driving frequency and lies inside the forbidden band. Additionally, as predicted by simulations [15,16], the DB appears to be long-lived, as shown for instance from the different decay rates in Figs. (4a) and (4b) after the actuator is switched off. In Fig. (4e), we estimate the PSD of the tail, illustrating that the DB maintains its prominence while the mode at the actuator frequency has experienced a decrease in PSD amplitude by two orders of magnitude.

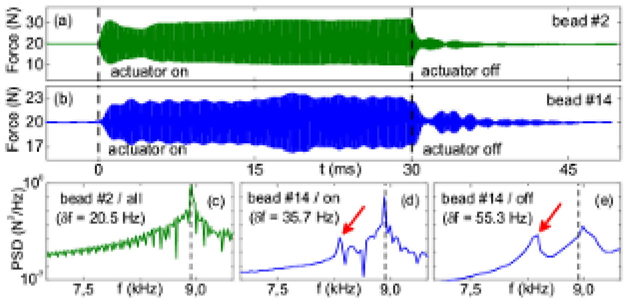

Figure 4: Experimental observation of a DB near the lower optical cutoff frequency. (a) Force at particle n=2, (b) force at particle n=14, (c) PSD at n = 2, (d) PSD at n = 14 while the actuator is on, and (e) PSD at n = 14 after the actuator is switched off. Vertical lines in (c,d,e) indicate the driving frequency. Arrows in (d,e) indicate a DB that lies inside the band gap, between the acoustic and the optical pass bands.

3 Conclusion

We characterized in [15] and [16] the dynamics of compressed 1D diatomic granular crystals using theory, numerical simulations, and experiments. We explored the mechanism leading to the formation of discrete breather via modulational instability, and provided clear experimental proof of their existence.

Acknowledgments

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