Nonlinearity Management in Optics and Bose-Einstein Condensation

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Outline

- Introduction
  - Nonlinear Schrödinger (NLS) equation
  - Nonlinearity management
- Pulse propagation in layered optical media
  - Arresting blow-up/collapse
  - Modulational instability
- Connections to Bose-Einstein condensation
- Conclusions

Nonlinear Schrödinger Equation

- NLS: $iu_z = -\Delta u - n_2|u|^2u$
  - $n_2 > 0$: focusing
  - $n_2 < 0$: defocusing
  - Integrable in 1 + 1 dimensions ($\Delta = d^2/dx^2$)
- Models wave envelopes in nonlinear dispersive media
- Nonlinear optics: Describes beam propagation in nonlinear optics incorporating dispersive and Kerr effects
- Bose-Einstein condensation: Describes mean-field dynamics at zero temperature
- Focusing NLS in d + 1 dimensions ($d \geq 2$) exhibits blow-up/collapse of pulse solutions

Nonlinearity Management

- NM = periodic variation in nonlinearity coefficient
- Optics: Stabilize pulses using layered media
  - Piecewise constant nonlinearity
  - Our work: First experimental implementation of NM + accompanying analysis and direct numerical simulations
- BEC: Use Feshbach resonances to vary interatomic interactions g and hence nonlinearity coefficient
  - Can achieve g = g(t) in numerous labs
  - New idea: periodic g = g(x) via "collisionally inhomogeneous" condensates (see our recent preprint, nlin.PS/0607009)
- Mathematical analyses via Hamiltonian-averaged NLS equations

Theoretical and Experimental Frameworks

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$$i\frac{\partial u}{\partial \zeta} = -\frac{1}{2}\nabla_\perp^2 u - |u|^2 u\,, \quad 0 < \zeta < \tilde{l} \quad \text{(glass)}\,,$$

$$i\frac{\partial u}{\partial \zeta} = -\frac{1}{2}\frac{n_0^{(1)}}{n_0^{(2)}}\nabla_\perp^2 u - \frac{n_2^{(2)}}{n_2^{(1)}}|u|^2 u\,, \quad \tilde{l} < \zeta < \tilde{L} \quad \text{(air)}$$

- u = scaled electric field envelope
- ζ = scaled propagation distance
- l = glass length (1 mm)
- L – l = air length
  - 1 mm, 1.5 mm, 2 mm

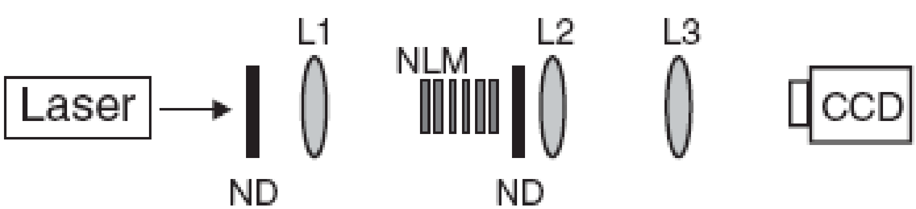


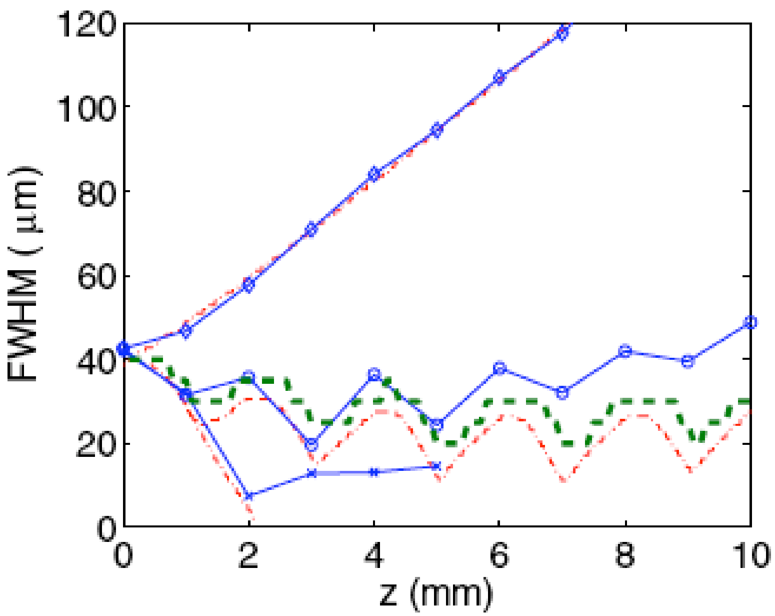
FIG. 1. Experimental setup. ND = neutral density filters, NLM = nonlinear medium, and L1–L3 = lenses.

- Laplacian: 2 dimensions
- Nonlinearity vs dispersion:
  - $n_2^{(2)}/n_2^{(1)} = 0.0001$
  - $n_0^{(1)}/n_0^{(2)} = 1.5$
- NLS simulations using Gaussians from experimental initial conditions (and experimental losses from reflection at slide interfaces)
  - No fitting parameters!
- Moment approach yields ODE whose solutions give qualitative coarse-grained dynamics

Delaying Blow-up and Collapse

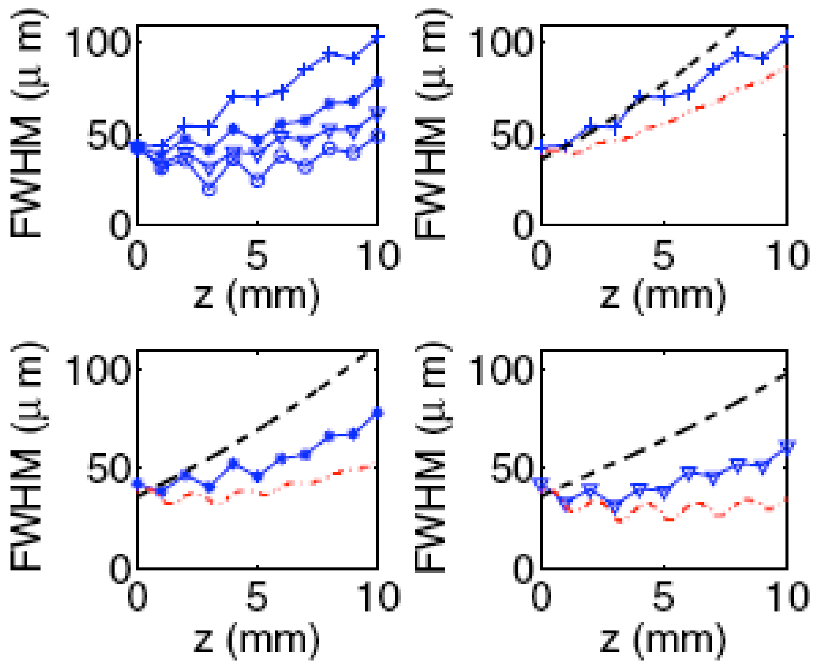
- Plot: Beam width versus propagation distance (P = 5.9 P_c)
  - Diverges in air
  - Collapses in glass
  - Can propagate in layered media much longer before divergence occurs

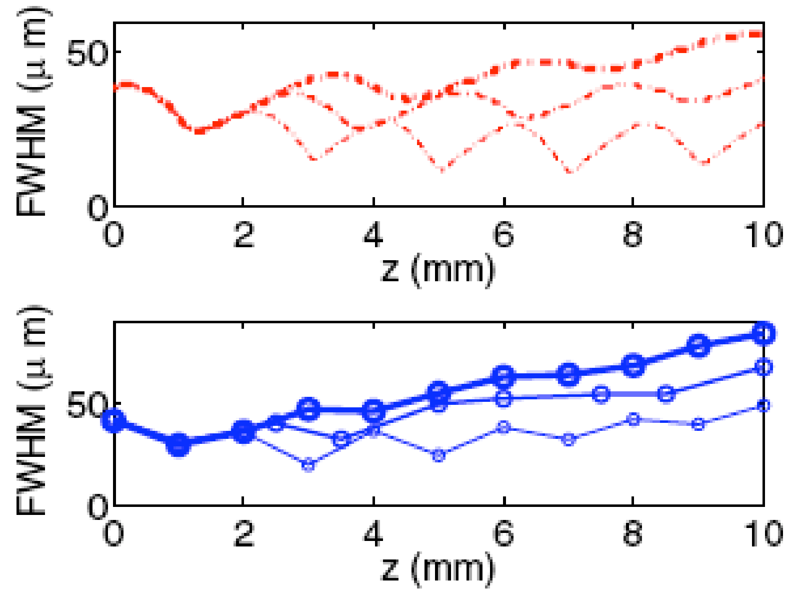
- Can arrest blow-up/collapse by alternating focusing and defocusing material (has not been done experimentally)



- 1 mm air gaps
- Blue curves: experiment
- Red and green curves: NLS simulations

Different powers and air gaps





- $P = 2.3P_c$ (+, top right),
$P = 3.9P_c$ (*, bottom left), $P = 4.9P_c$ (∇, bottom right), $P = 5.9P_c$ (°)
- Black curves: ODE theory
- 1 mm air gaps

- 1 mm air gaps (thin), 1.5 mm gaps (medium), 2 mm gaps (thick)
- Top: NLS simulations
- Bottom: Experiments
- $P = 5.9P_c$

Modulational Instability

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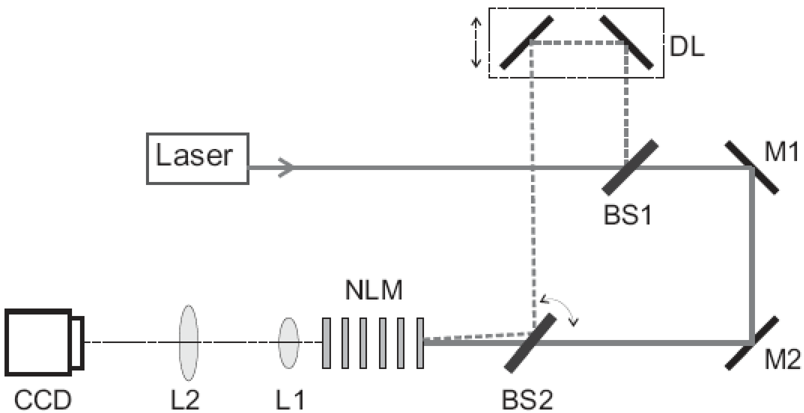


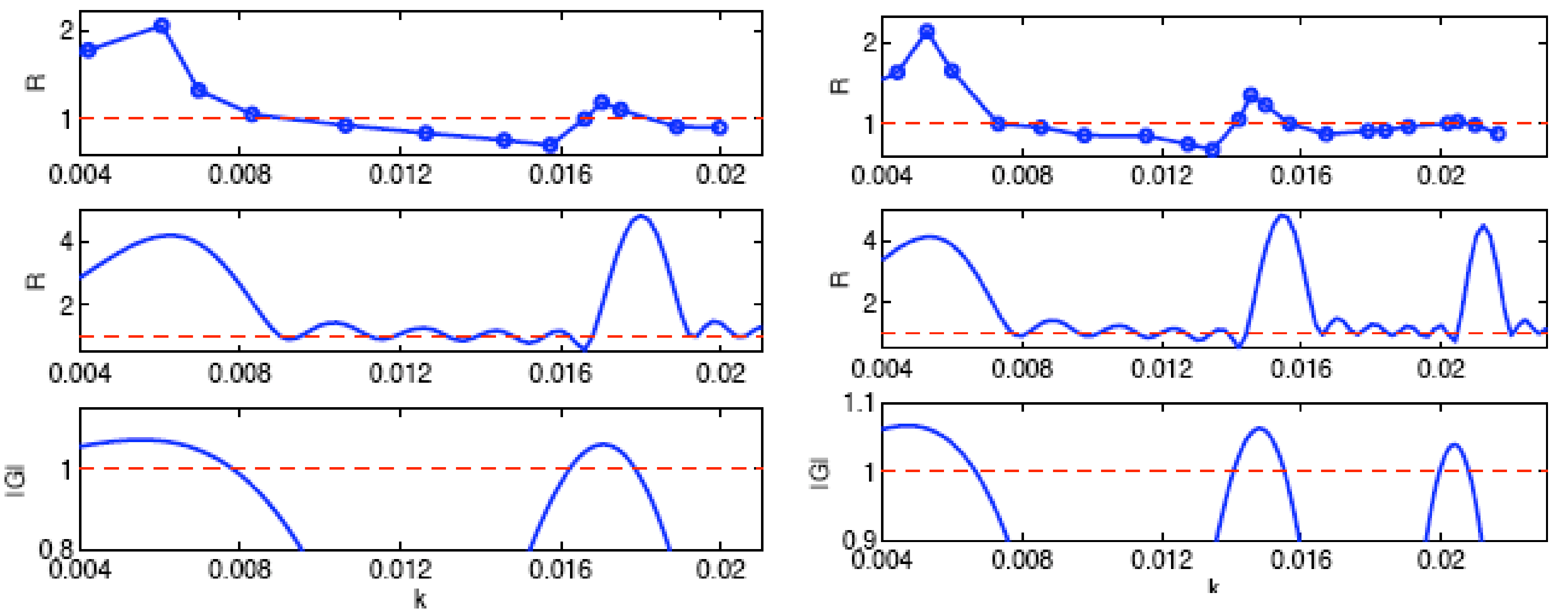
$$i\frac{\partial u}{\partial \zeta} = -\frac{1}{2}\nabla_\perp^2 u - |u|^2 u\,, \quad 0 < \zeta < \tilde{l} \quad (\text{glass})\,,$$

$$i\frac{\partial u}{\partial \zeta} = -\frac{1}{2}\frac{n_0^{(1)}}{n_0^{(2)}}\nabla_\perp^2 u - \frac{n_2^{(2)}}{n_2^{(1)}}|u|^2 u\,, \quad \tilde{l} < \zeta < \tilde{L} \quad (\text{air})$$

- 1 mm glass slides
- 2.1 mm or 3.1 mm air gaps
- reflective coating on slides to reduce losses at interfaces

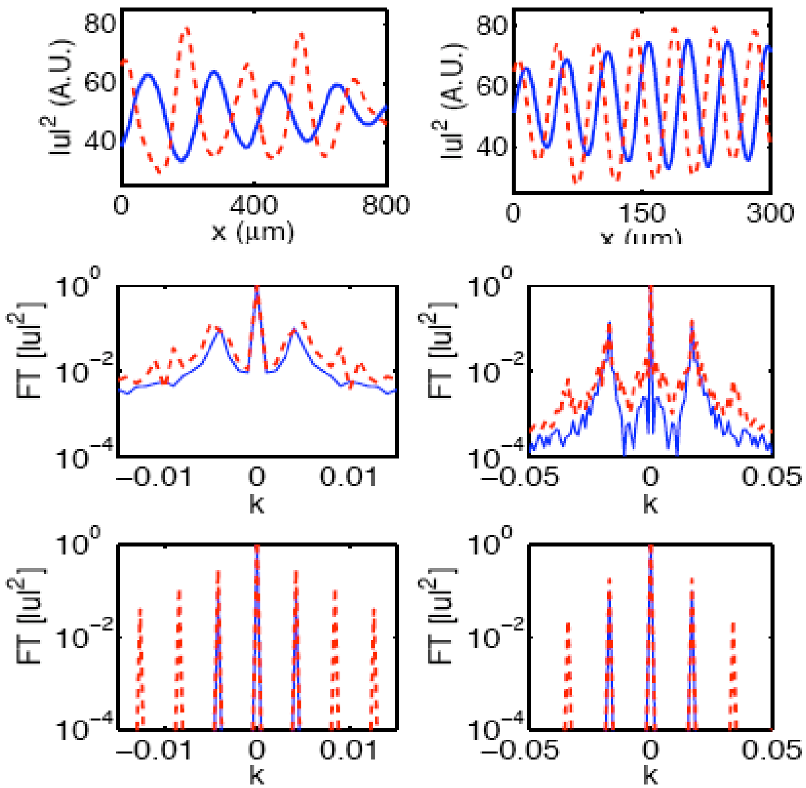
- MI = destabilization mechanism for plane waves due to interplay between nonlinearity and dispersion
  - ⇒ formation of localized pulses
- Arises ubiquitously; in fluid dynamics ("Benjamin-Feir" instability), nonlinear optics, plasma physics, BEC,etc.
- In uniform media, focusing nonlinearity ⇒ MI for sufficiently large plane-wave amplitudes (given the wavenumber) or sufficiently small wavenumbers (given the amplitude)
  - i.e.,1 instability band

"New Physics": Extra MI Bands

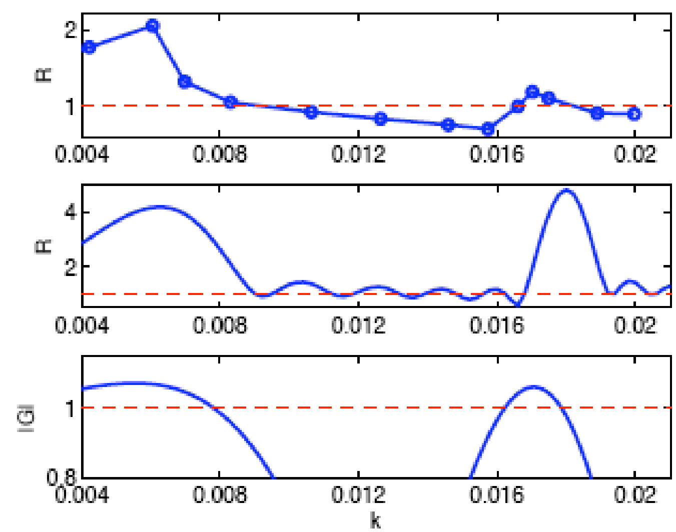


- Periodicity in propagation variable ⇒ multiple MI bands (R, lGl > 1)
- *Quantitative* agreement in band locations (0 fitting parameters!)
- Top: Experiments. Middle: NLS simulations. Bottom: Analysis
- Left: 2.1 mm air gaps. Right: 3.1 air gaps.
- R = perturbation growth (measured using relative sizes of Fourier peaks of wavenumbers); lGl comes from Kronig-Penney equation

MI II: Fourier Peaks



- 2.1 mm air gaps
- Left: First MI band. Right: Second MI band.
- Red: High intensity. Blue: Low intensity.



- Top: Experiments (intensity)
- Middle: Experiments (Fourier transform)
- Bottom: NLS simulations (Fourier transform).

MI III: Linear Stability Analysis

$$i\frac{\partial u}{\partial \zeta} = -\frac{1}{2}D(\zeta)\nabla^2 u - N(\zeta)|u|^2 u - i\gamma(\zeta)u \qquad \gamma(\zeta) = \alpha\sum_{n=1}^{M}\delta(\zeta - \zeta_n)$$

- D(ζ), N(ζ): management functions
  - Here: piecewise constant
- γ(ζ): losses at glass-air interfaces
- Plane wave solutions:

$$u_0 = A_0 e^{-\int^{\zeta}\gamma(\zeta')d\zeta'} e^{iA_0^2\int^{\zeta} N(\zeta')\left(e^{-2\int^{\zeta'}\gamma(\tilde{\xi})d\tilde{\xi}}\right)d\zeta'}$$

- Perturb from plane waves: $u = u_0(\zeta)\left[1 + w(\zeta)\cos(k_{\xi}\xi)\cos(k_{\eta}\eta)\right]$

MI III: Linear Stability Analysis

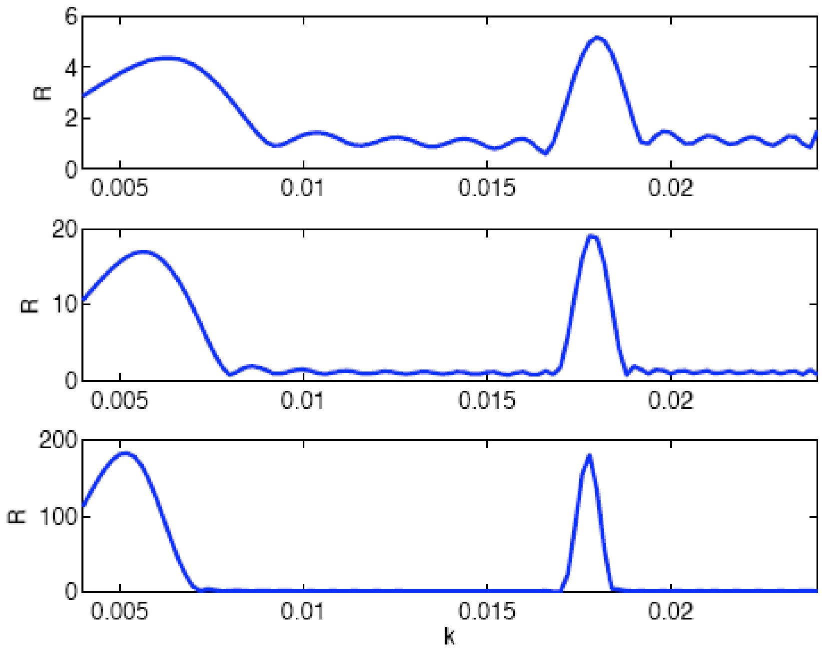
$$\frac{d^2F}{d\zeta^2} = \frac{1}{D(\zeta)}\frac{dD}{d\zeta}(\zeta)\frac{dF}{d\zeta} + \left[-\frac{1}{4}\bar{k}^4 D(\zeta)^2 + N(\zeta)\bar{k}^2 D(\zeta)|u_0(\zeta)|^2\right] F$$

- $w = F + iB$, $k^2 = k_\xi{}^2 + k_\eta{}^2$
- $F = gD^{1/2} \Rightarrow$ Hill equation
  - $\Rightarrow$ Can apply Floquet-Bloch theory
- Piecewise constant coefficients $\Rightarrow$ Kronig-Penney equation
  - $\Rightarrow$ Can solve for MI bands analytically!

$$\cos(\omega\tilde{L}) = -\frac{s_1^2 + s_2^2}{2s_1 s_2}\sin(s_1\tilde{l})\sin[s_2(\tilde{L}-\tilde{l})] + \cos(s_1\tilde{l})\cos[s_2(\tilde{L}-\tilde{l})] \equiv G(\bar{k})$$

- ω is Floquet multiplier; s_1, s_2 expressed in terms of D, N, k, $|u_0|$
- $|G(k)| > 1 \Rightarrow$ MI

MI IV: More Propagation Periods



- Top: 6 glass, 5 air
  - Experimental configuration
- Middle: 11 glass, 10 air
- Bottom: 21 glass, 20 air

- Perturbations in MI bands grow exponentially but those outside (i.e., the ripples) saturate
- ⇒ Slight discrepancies in numerics vs experiments and theory (uses ∞ periods) due to finite number of propagation periods

Connections to Bose-Einstein Condensation

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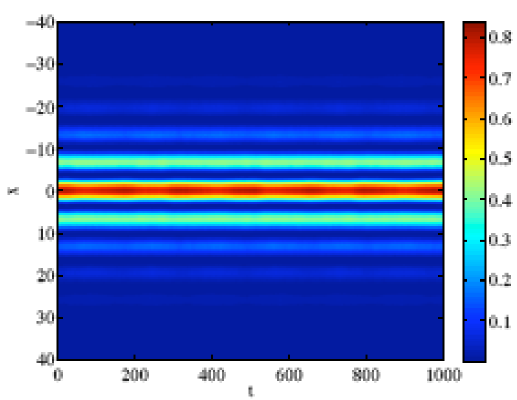
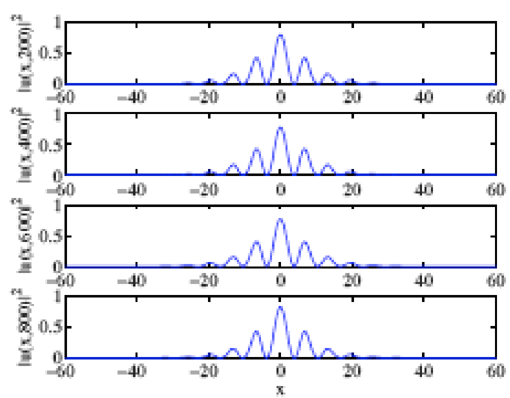
- Feshbach resonances can be used for nonlinearity management in BEC
  - Hamiltonian-average over periodic adjustment in scattering length [g = g(t)] to get an effective NLS:

$$iu_t + u_{xx} = \epsilon\left(2\cos(\omega x)u + \gamma_0|u|^2u - \gamma_1^2\left(\left(\left(|u|^2\right)_x\right)^2 + 2|u|^2\left(|u|^2\right)_{xx}\right)u\right)$$

  - We construct solitary wave solutions ("gap solitons") and study their stability.





Conclusions

- General theme: Interactions between nonlinearity & periodicity
- Layered optical media
  - First experimental implementation of nonlinearity management
  - Can delay blow-up/collapse using two focusing media (e.g., glass and air) with method that is lossless in principle
    - Theory: Can prevent it by alternating focusing and defocusing media
  - Very good agreement with NLS simulations (zero fitting parameters!) and coarser features captured by a simplified ODE framework
- Modulational instability: theory, NLS simulations, and experiments give excellent *quantitative* agreement for locations of instability bands (zero fitting parameters!)
  - New physics: Only one band in uniform media but multiple bands in layered media
- Connections to Feshbach resonance management in Bose-Einstein condensation