Math Ma **Derivative of Logarithms** Fall 2016

1. Let $f(x) = \ln x$.

   (a) Use the limit definition to find $f'(x)$, the derivative of $\ln x$.

   (b) Sketch the graphs of $f(x)$ and $f'(x)$.

   (c) Sketch the graphs of e^x and $\ln x$ on the same set of axis. Then find $f'(x)$, the derivative of $\ln x$.

   (d)

x	estimate of $f'(x)$
0.1	9.53
0.5	1.98
1	0.995
2	0.499
3	0.333
4	0.250
5	0.200

   What do you observe from the table?

2. What is the derivative of $\log_2 x$? (Hint: Use the change of base formula.)

3. Find the derivative of each of the following.

(a) $f(x) = x \ln x$

(b) $f(x) = \frac{\ln(3x)}{x}$

(c) $f(x) = \log_3 \sqrt{3x} + \sqrt{x}$

(d) $f(x) = \log_5 \frac{1}{x^2}$

(e) $f(x) = \log_2 3^x$

(f) $f(x) = 5^{\log_7 x}$

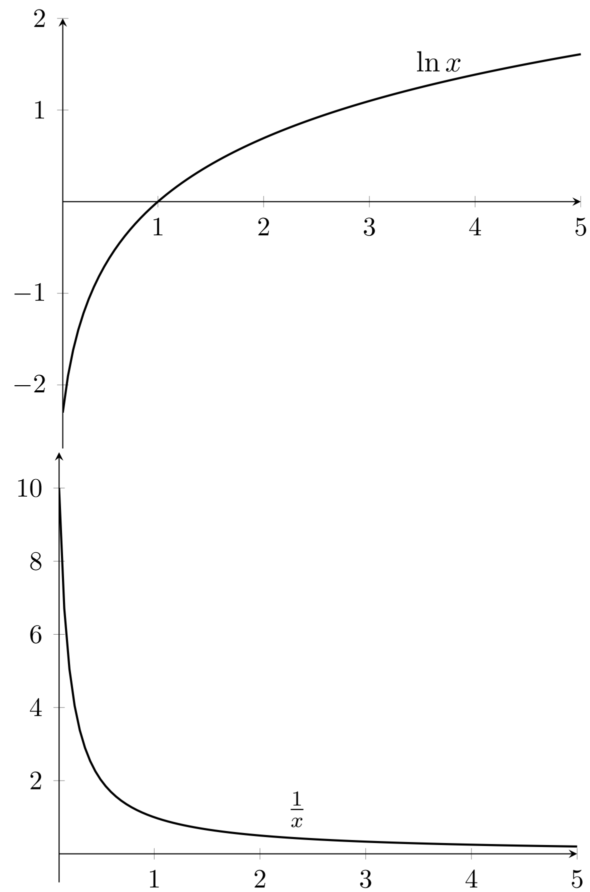
Derivative of Logarithms – Solutions

1. (a)

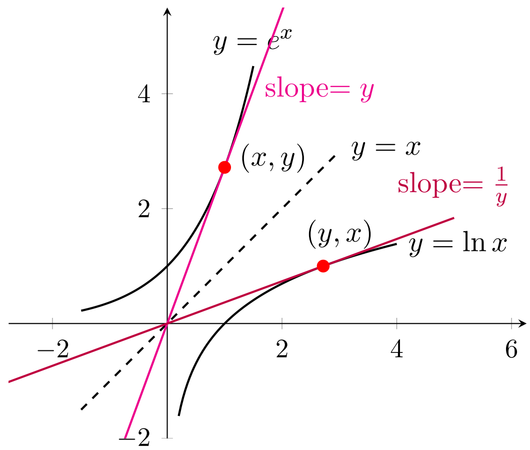
$$f'(x) = \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h} = \lim_{h\to 0} \frac{\ln \frac{x+h}{x}}{h} = \lim_{h\to 0} \frac{\ln\left(1+\frac{h}{x}\right)}{h} = \lim_{h\to 0} \frac{\ln\left(1+\frac{h}{x}\right)}{\frac{h}{x}} \cdot \frac{1}{x}$$

$$= f'(1) \cdot \frac{1}{x} = \frac{1}{x}$$

(b)



(c)



(d) $f'(x) = \frac{1}{x}$.

2. By change of base formula,

$$\log_2 x = \frac{\ln 2}{\ln x}.$$

Then using constant multiple rule,

$$(\log_2 x)' = \left(\frac{\ln x}{\ln 2}\right)' = \frac{1}{\ln 2} \cdot (\ln x)' = \frac{1}{\ln 2 \cdot x}.$$

3. (a) Using product rule,

$$(x \ln x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

(b) First of all, $f(x) = \frac{\ln 3 + \ln x}{x}$. Hence usnig quotient rule

$$\left(\frac{\ln 3 + \ln x}{x}\right)' = \frac{\frac{1}{x} \cdot x - (\ln 3 + \ln x) \cdot 1}{x^2} = \frac{1 - \ln 3 - \ln x}{x^2}.$$

(c) First of all, $f(x) = \frac{1}{2} + \frac{1}{2} \log_3 x + \sqrt{x}$, so

$$f'(x) = \frac{1}{2 \ln 3 \cdot x} + \frac{1}{2\sqrt{x}}.$$

(d) First of all, $f(x) = -2 \log_5 x$, so

$$f'(x) = \frac{-2}{\ln 5 \cdot x}.$$

(e) First of all, $f(x) = x \cdot \log_2 3$, so

$$f'(x) = \log_2 3.$$

(f) First of all, $f(x) = 5^{\frac{\log_5 x}{\log_5 7}} = x^{\frac{1}{\log_5 7}}$, so using power rule

$$f'(x) = \frac{1}{\log_5 7} \cdot x^{\frac{1}{\log_5 7} - 1}.$$